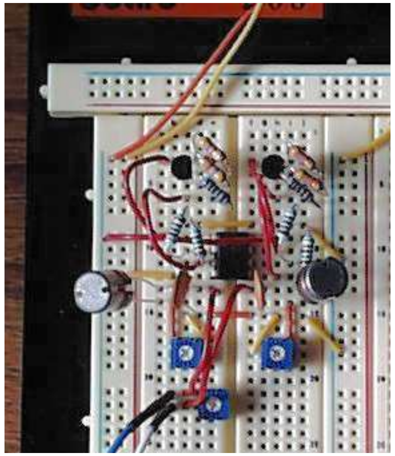
Bifurcation and Chaos in Asymmetrically Coupled BVP Oscillators



T. Ueta and H. Kawakami
Tokushima University, Japan

Brief history of BVP (Bohnhöffer van der Pol) oscillator

- A 2nd-dim sytem derived from Hodkin-Huxley (HH) equation.
- FitzHugh-Nagumo oscillator, extracting excitatory behavior of HH equation.
- Nonlinearity: only a cubic term is included.

Circuit realization BVP oscillator is a natural extension of van der Pol oscillator. 日本語日本語

- evaluate internal impedance of a coil
- add a bias power source to remove symmetry of the origin

A. N. Bautin, “Qualitative investigation of a Particular Nonlinear System,” PPM, 1975. $\longrightarrow$ a detail topological classification of BVP equation

Coupled BVP oscillators

Coupled symmetrical BVP oscillators system have been studied by using the group theory.

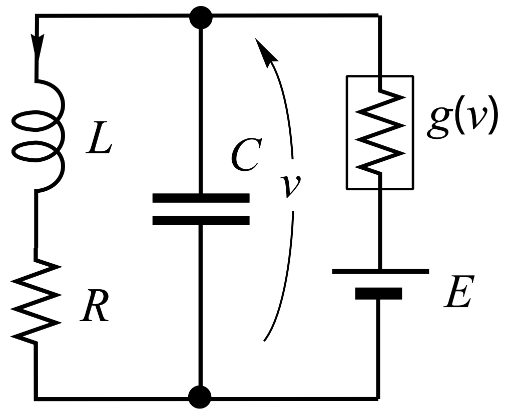
$$\Downarrow$$

No chaotic behavior is found in real circuitry

This presentation shows ...

- asymmetrically coupled BVP oscillators
- circuit configuration and equations
- bifurcaiton phenomena of equilibria and limit cycles
- results of laboratory experiments

Single BVP Oscillator

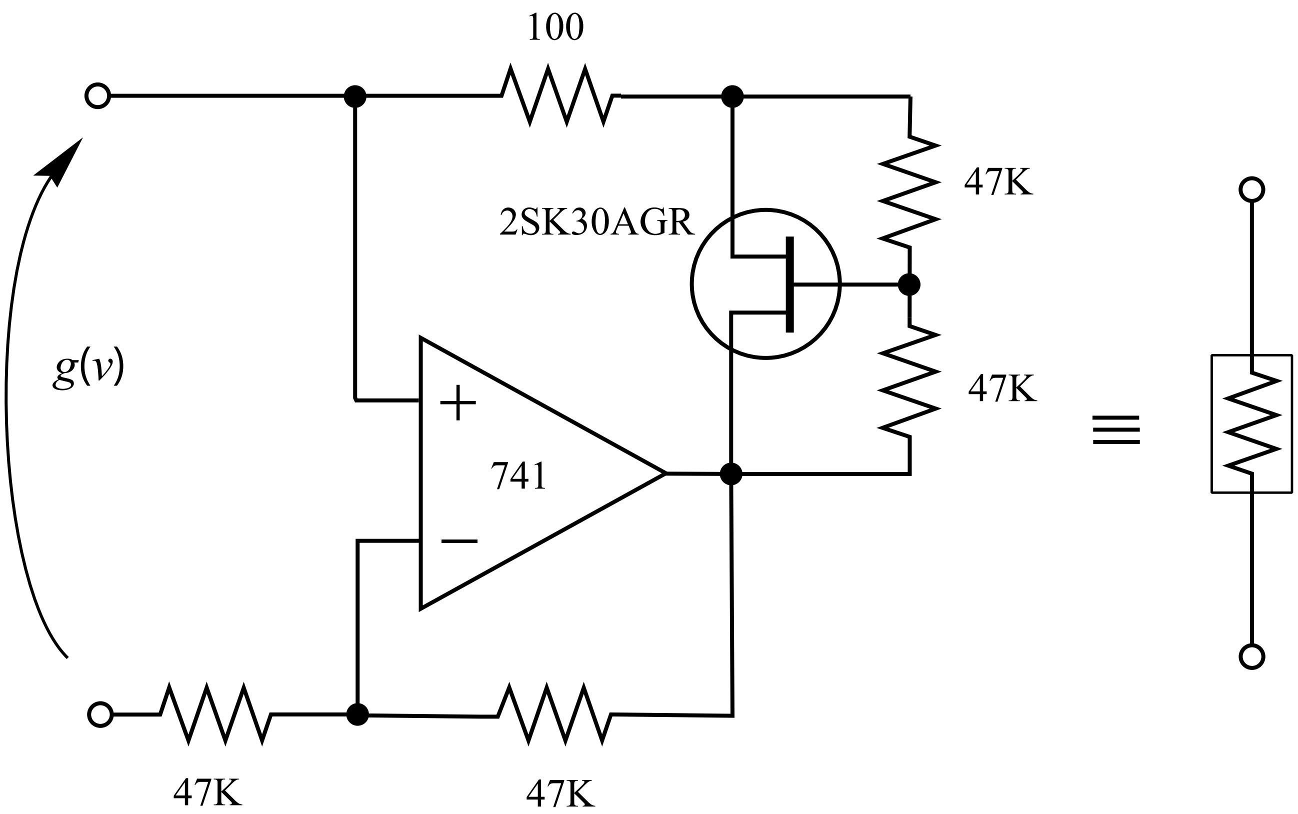


$$C\frac{dv}{dt} = -i - g(v)$$

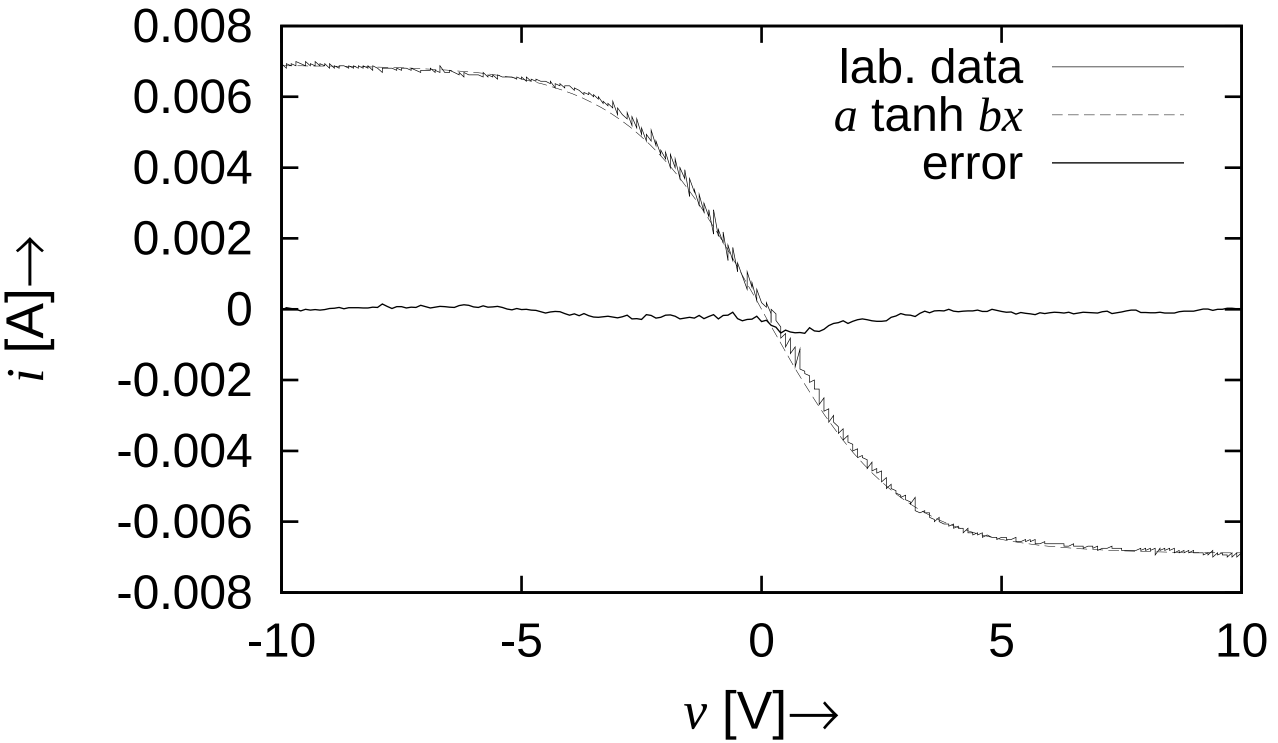
$$L\frac{di}{dt} = v - ri + E$$

There exist two port to extract state variables v and i.

Nonlinear resister
with 2SK30A FET:



Measurement of the nonlinear resister



$g(v) = -a \tanh bv$ **with** $a = 6.89099 \times 10^{-3}$, $b = 0.352356$.

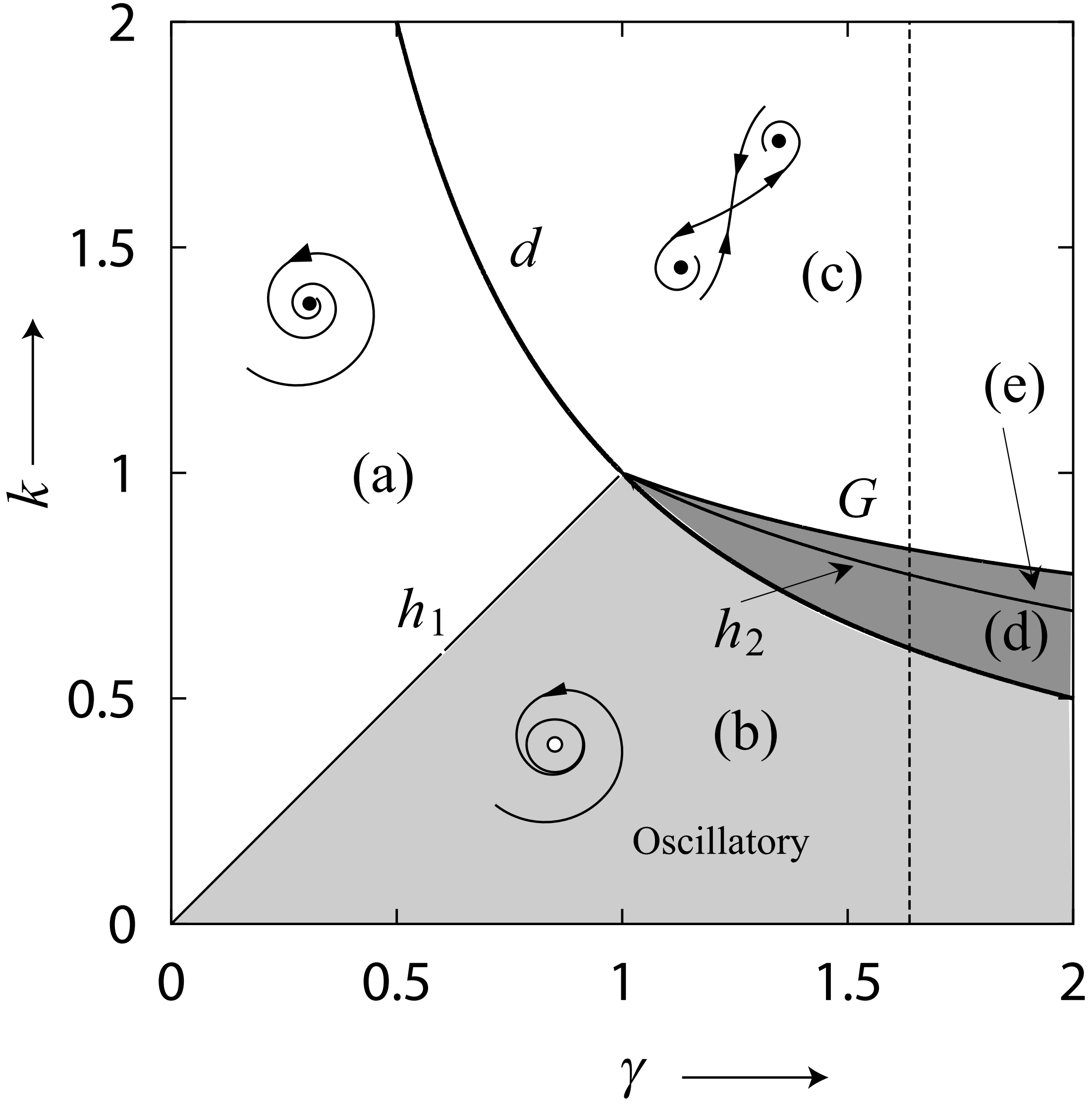
BVP equation

$$\begin{aligned}\dot{x} &= -y + \tanh \gamma x \\ \dot{y} &= x - ky.\end{aligned}$$

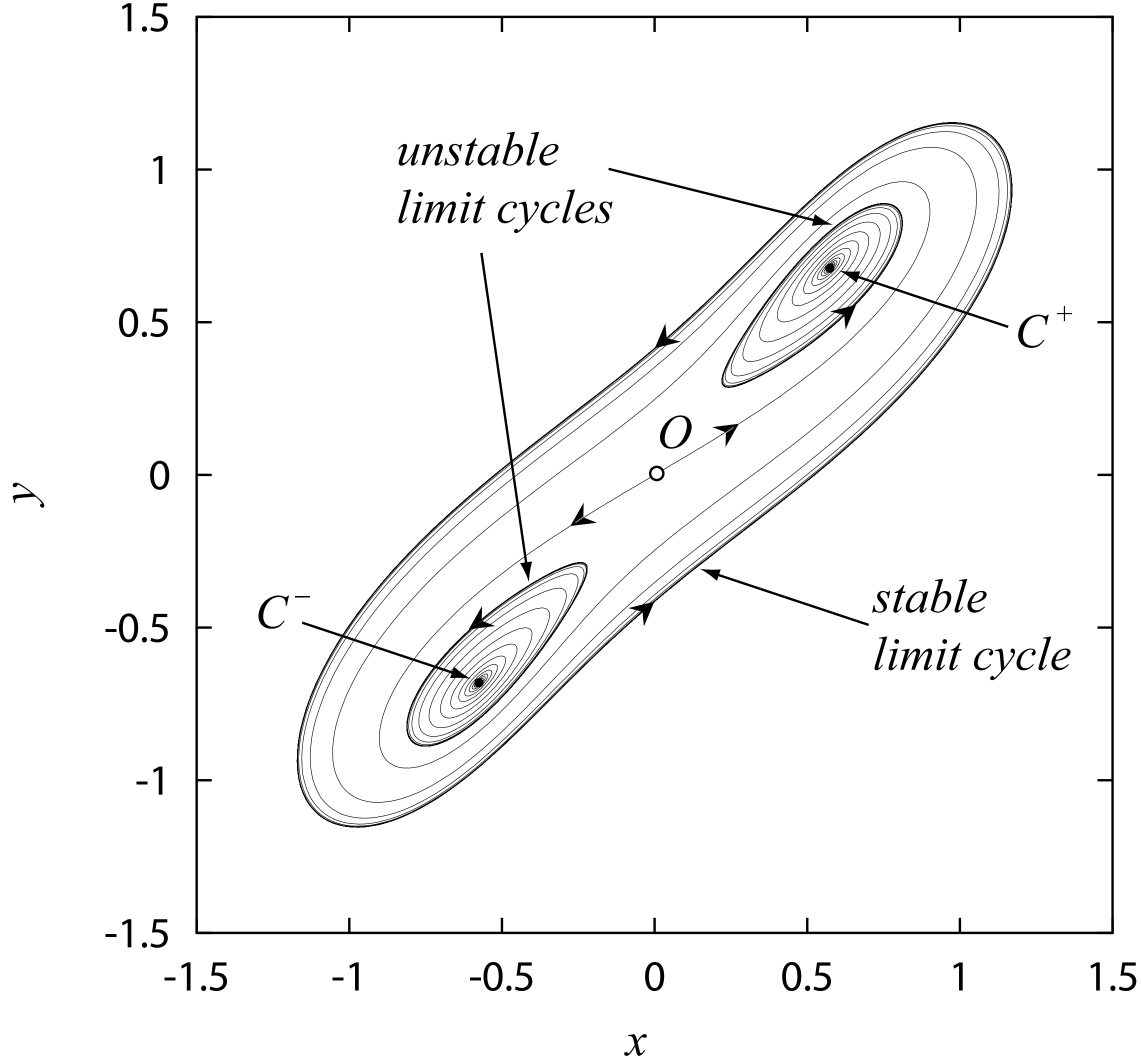
$$\cdot = d/d\tau, \qquad x = \sqrt{\frac{C}{L}}v, \qquad y = \frac{i}{a}$$

$$\tau = \frac{1}{\sqrt{LC}}t, \qquad k = r\sqrt{\frac{C}{L}}, \qquad \gamma = ab\sqrt{\frac{L}{C}}$$

Bifurcation of Single BVP oscillator



An example flow in area (d)

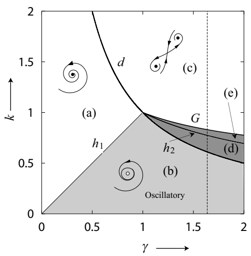


Circuit parameters

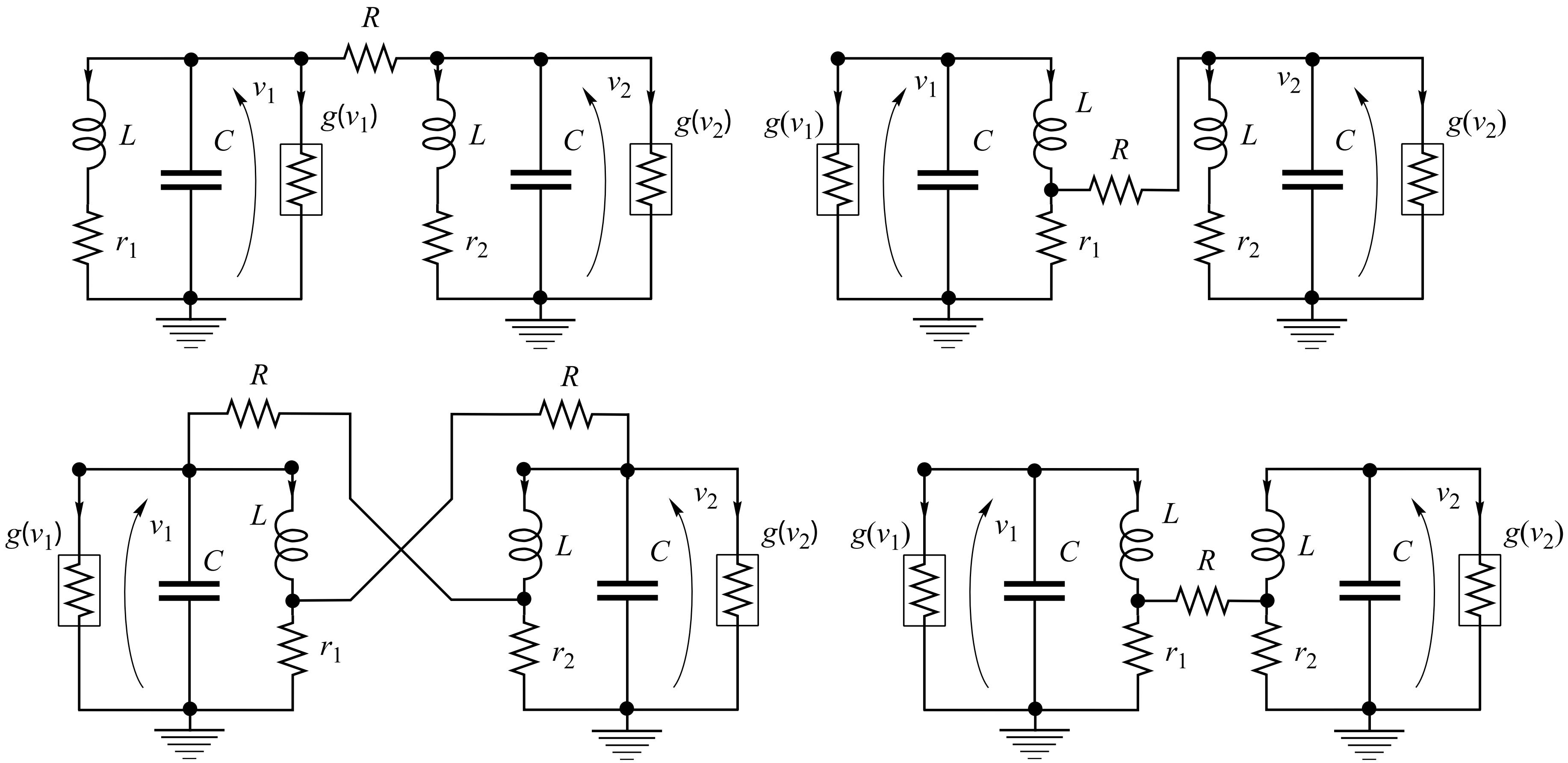
$$L = 10 \textbf{ [mH]}, \quad C = 0.022 \textbf{ [}\mu\textbf{F]}$$

$$\Downarrow$$

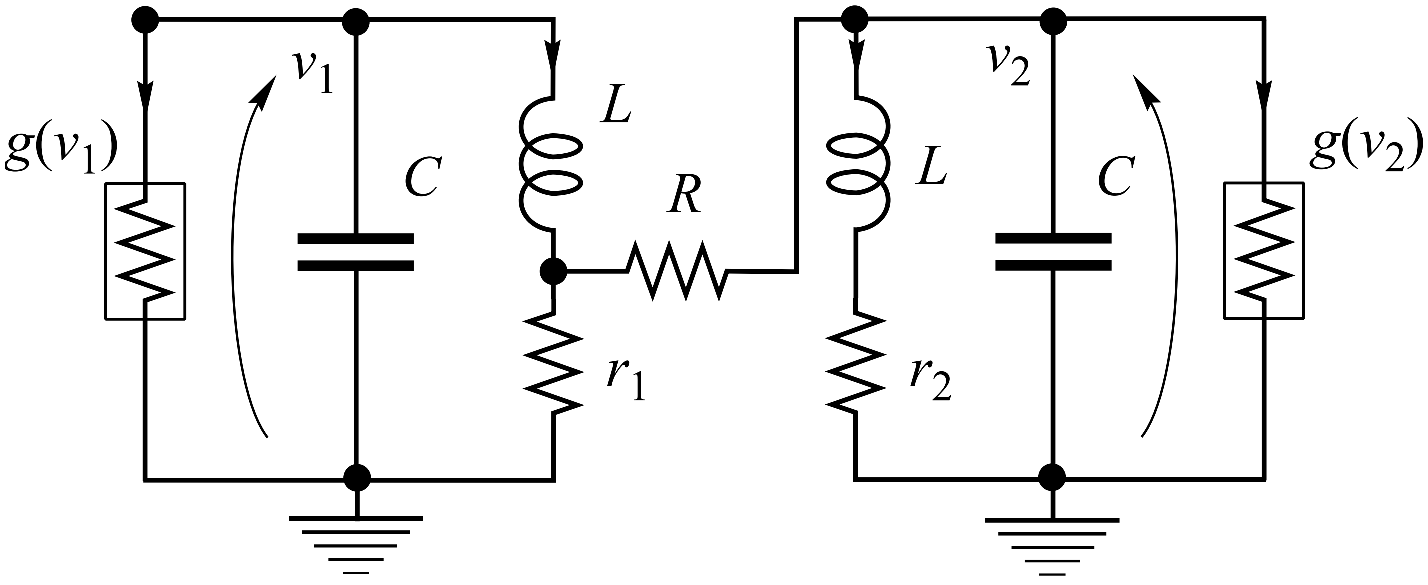
$$\gamma = 1.6369909, \sqrt{\frac{L}{C}} = 674.19986.$$



Resistively coupled BVP oscillators



Asymmetrically coupled BVP oscillators



Circuit equations

$$C\frac{dv_1}{dt} = -i_1 - g(v_1)$$

$$L\frac{di_1}{dt} = v_1 - r_1 i_1 + \frac{Gr_1}{1+Gr_1}(r_1 i_1 - v_2)$$

$$C\frac{dv_2}{dt} = -i_2 - g(v_2) + \frac{G}{1+Gr_1}(r_1 i_1 - v_2)$$

$$L\frac{di_2}{dt} = v_2 - r_2 i_2$$

Normalized equations

$$x_j = \frac{v_j}{a}\sqrt{\frac{C}{L}}, \qquad y_j = \frac{i_j}{a}, \qquad k_j = r_j\sqrt{\frac{C}{L}}, \qquad j = 1, 2.$$

$$\tau = \frac{1}{\sqrt{LC}}t, \qquad \gamma = ab\sqrt{\frac{L}{C}}, \qquad \delta = G\sqrt{\frac{L}{C}}.$$

$$\eta = \frac{1}{1 + \delta k_1}$$

Normalized equation (cont.)

$$
\begin{aligned}
\frac{dx_1}{d\tau} &= -y_1 + \tanh \gamma x_1 \\
\frac{dy_1}{d\tau} &= x_1 - k_1 y_1 + \delta k_1 \eta (k_1 y_1 - x_2) \\
\frac{dx_2}{d\tau} &= -y_2 + \tanh \gamma x_2 + \delta \eta (k_1 y_1 - x_2) \\
\frac{dy_2}{d\tau} &= x_2 - k_2 y_2
\end{aligned}
$$

Symmetry

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$$

where, $\boldsymbol{f} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ **:** C^∞ **for** $\boldsymbol{x} \in \boldsymbol{R}^n$**.**

$$P : \boldsymbol{R}^n \to \boldsymbol{R}^n$$
$$\boldsymbol{x} \mapsto P\boldsymbol{x}$$

P**-invariant equation:**

$$\boldsymbol{f}(P\boldsymbol{x}) = P\boldsymbol{f}(\boldsymbol{x}) \qquad \textbf{for all } \boldsymbol{x} \in \boldsymbol{R}^n$$

Matrix P

- **If $k_1 = k_2$, then**

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

group for product:
$\Gamma = \{P, -P, I_n, -I_n\}$

- **If $k_1 \neq k_2$ then $\Gamma = \{I_n, -I_n\}$**

Poincaré 写像解 $\varphi(t)$:

$$x(t) = \varphi(t, x_0), \qquad x(0) = x_0 = \varphi(0, x_0)\,.$$

Poincaré 切断面:

$$\Pi = \{\, x \in R^n \;\mid\; q(x) = 0 \,\}\,,$$

$$T : \widehat{\Pi} \to \Pi; \qquad \tilde{x} \mapsto \varphi(\tau(\tilde{x}), \tilde{x})\,,$$

周期解 $\varphi(t)$ について，固定点が対応する：

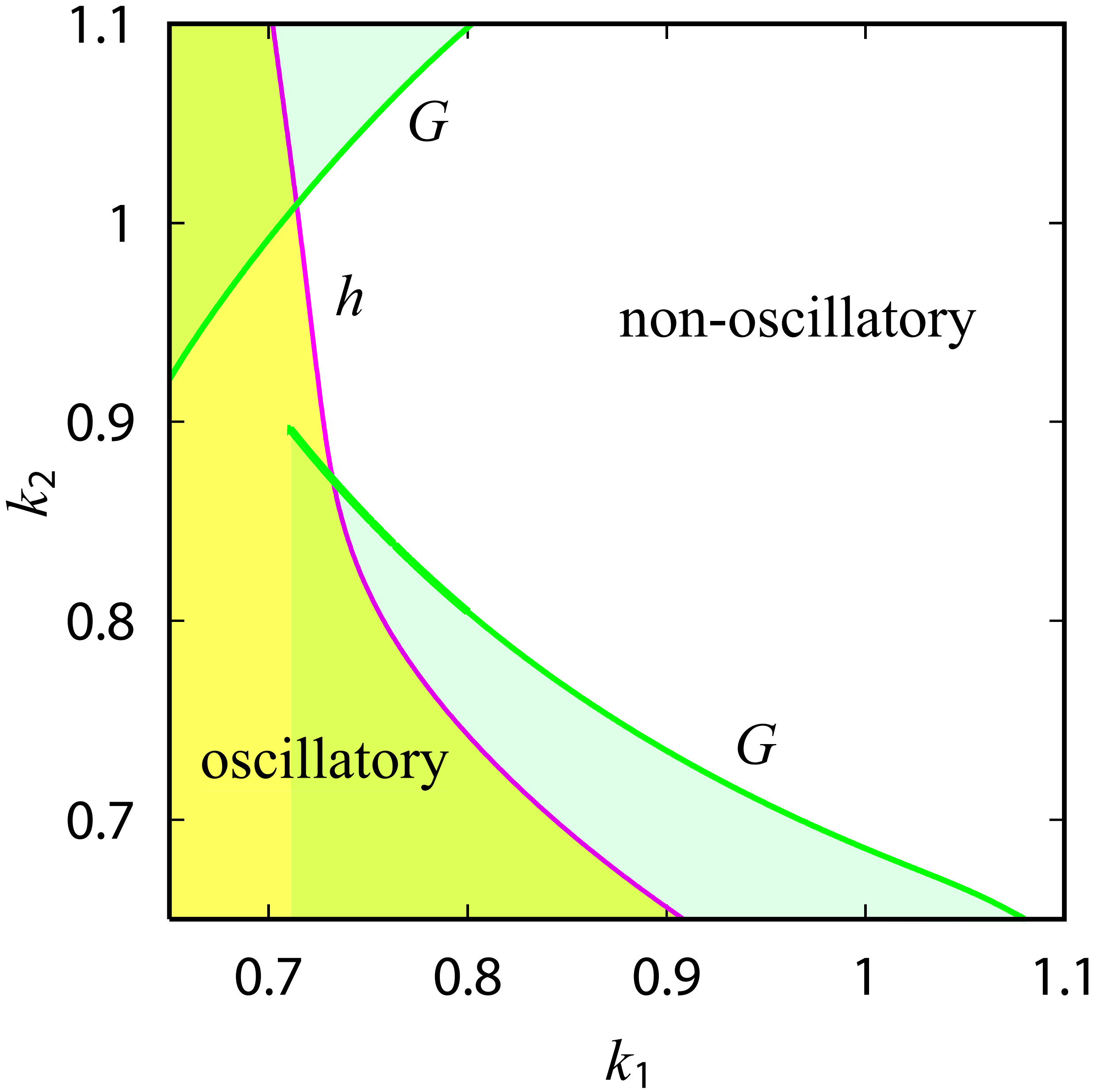
$$T(x_0) = x_0.$$

特性方程式と局所分岐

$$\chi(\mu) = \det\left(\frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{x}_0} - \mu I_n\right).$$

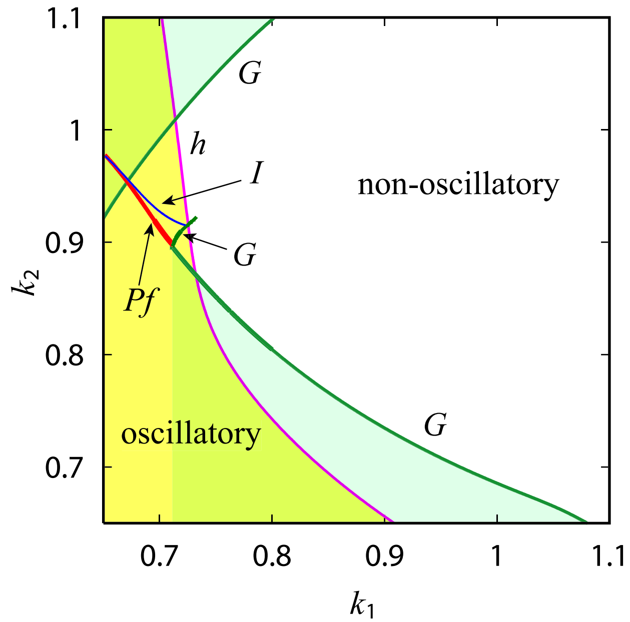
分岐の種類：

- $\mu = 1$: **接線分岐**
- $\mu = -1$: **周期倍分岐**
- $\mu = e^{j\theta}$: **Neimark-Sacker 分岐**
- **Pitchfork 分岐**



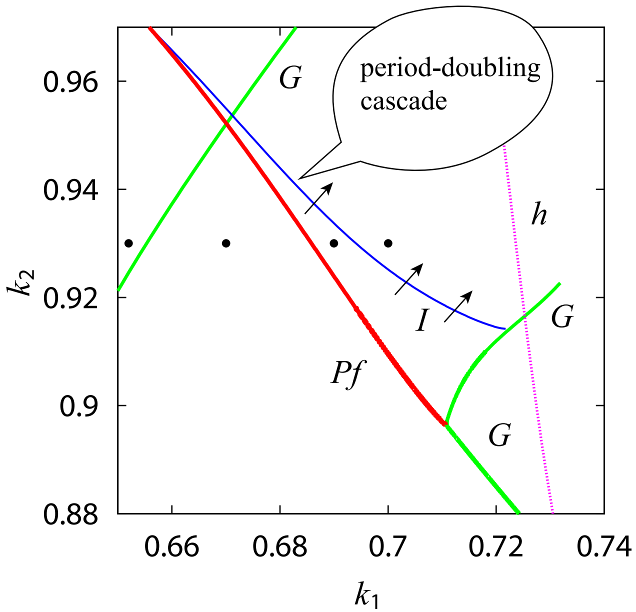
Bifurcation diagram

周期解の分岐図



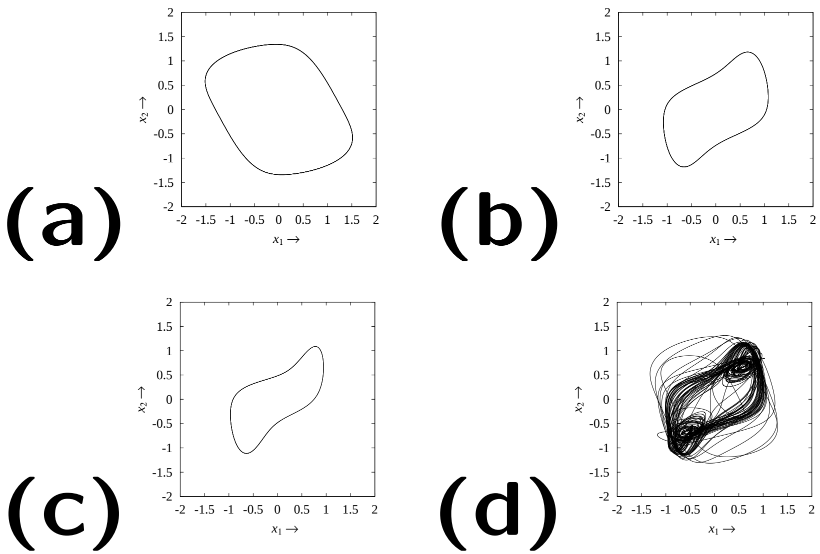
1.1
1
0.9
0.8
0.7
0.7
0.8
0.9
1
1.1
k_2
k_1
G
h
I
non-oscillatory
G
Pf
oscillatory
G

周期解の分岐図 (拡大図)

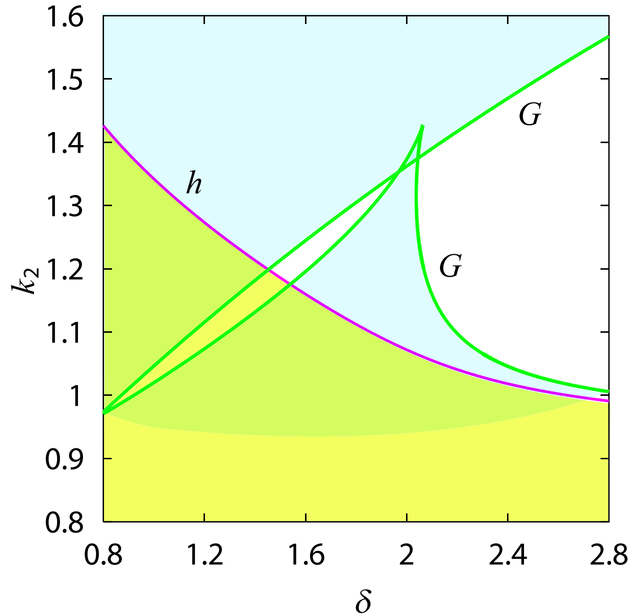


period-doubling
cascade
G
G
G
h
I
Pf
0.96
0.94
0.92
0.9
0.88
k_2
0.66
0.68
0.7
0.72
0.74
k_1

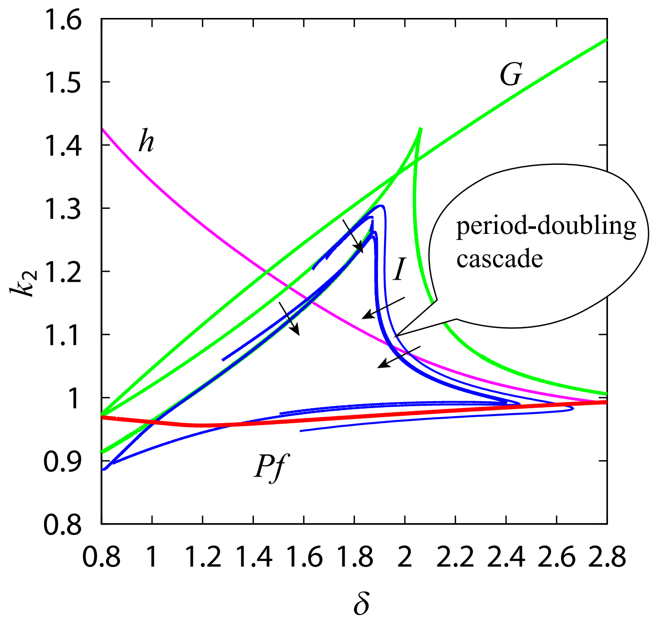
位相平面図(x_1-x_2)



平衡点・周期解の分岐図

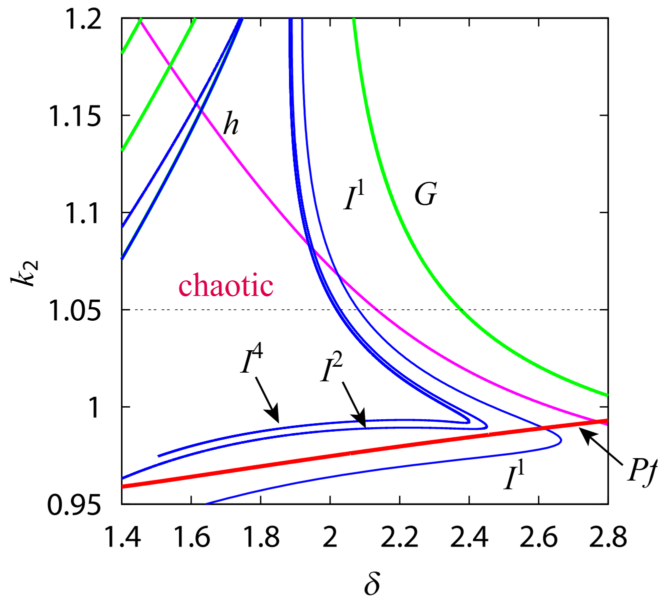


平衡点・周期解の分岐図

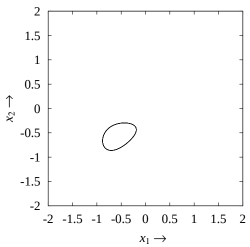


G
h
period-doubling
cascade
I
Pf
k_2
δ
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8

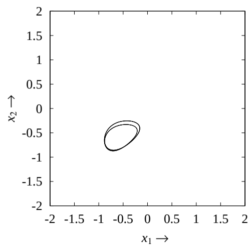
平衡点・周期解の分岐図(拡大図)



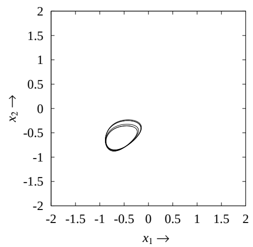
周期解の分岐



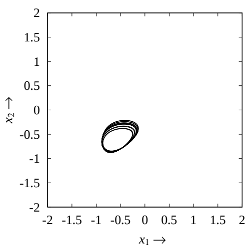
$x_2 \rightarrow$
$x_1 \rightarrow$



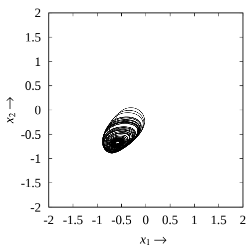
$x_2 \rightarrow$
$x_1 \rightarrow$



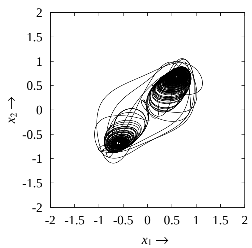
$x_2 \rightarrow$
$x_1 \rightarrow$



$x_2 \rightarrow$
$x_1 \rightarrow$

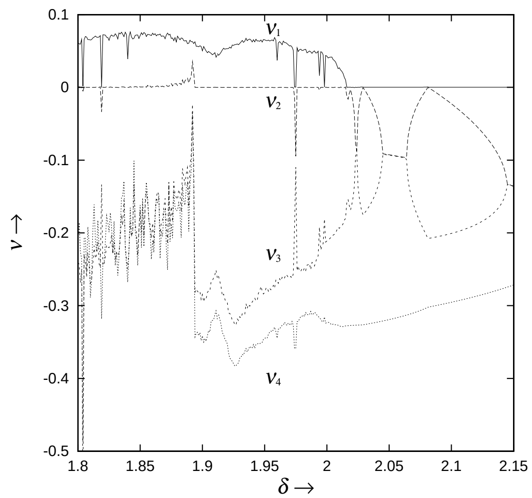


$x_2 \rightarrow$
$x_1 \rightarrow$



$x_2 \rightarrow$
$x_1 \rightarrow$

リアプノフ指数



ν_1
ν_2
ν_3
ν_4
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
$\nu \rightarrow$
$\delta \rightarrow$