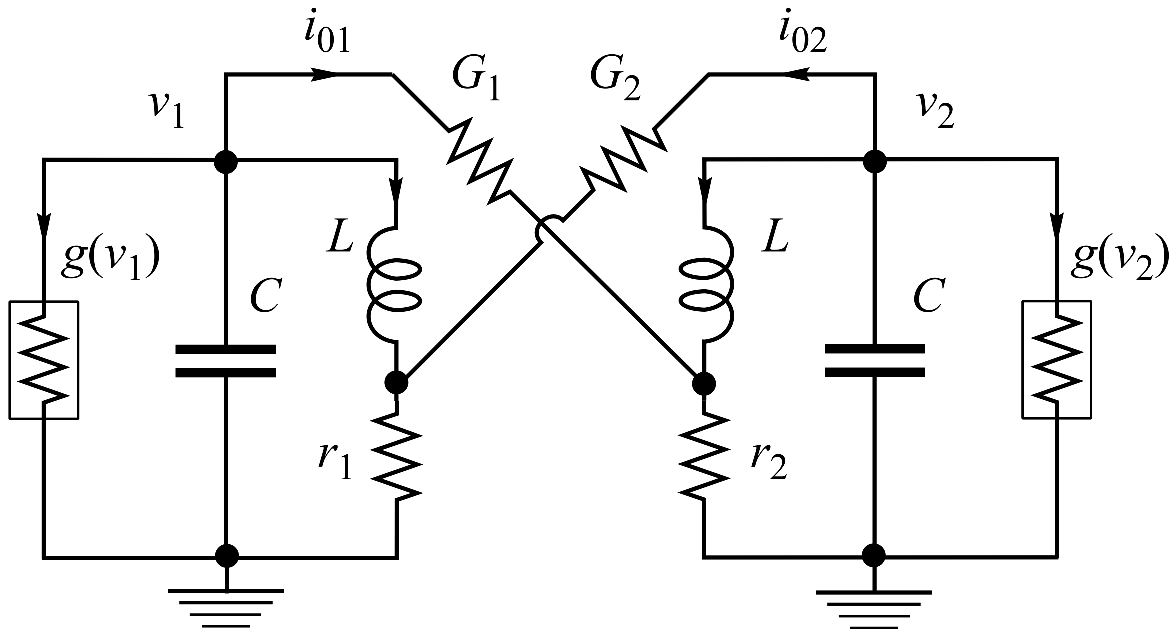
Chaos in Cross-Coupled BVP Oscillators



Tetsushi Ueta and Hiroshi Kawakami

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Nonlinear coupled oscillators

- power lines system
- neural networks
- biological activities

Nonlinear coupled oscillators

- ✎ **power lines system**
- ✎ **neural networks**
- ✎ **biological activities**

Decomposition a complex nonlinear dynamics into unit oscillators and their connections

Nonlinear coupled oscillators

- ✎ **power lines system**
- ✎ **neural networks**
- ✎ **biological activities**

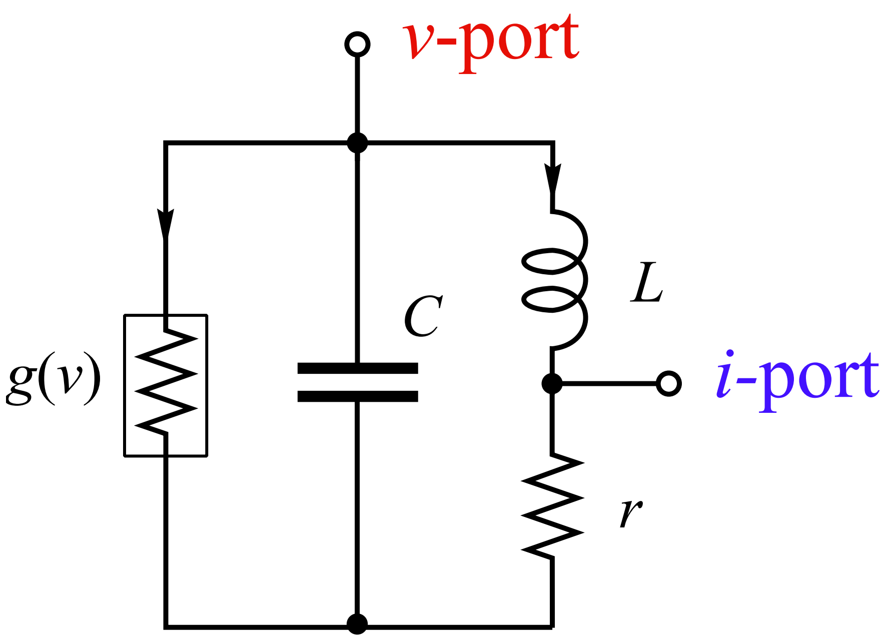
Decomposition a complex nonlinear dynamics into unit oscillators and their connections

$\Downarrow$

a reduced dynamical system with symmetry

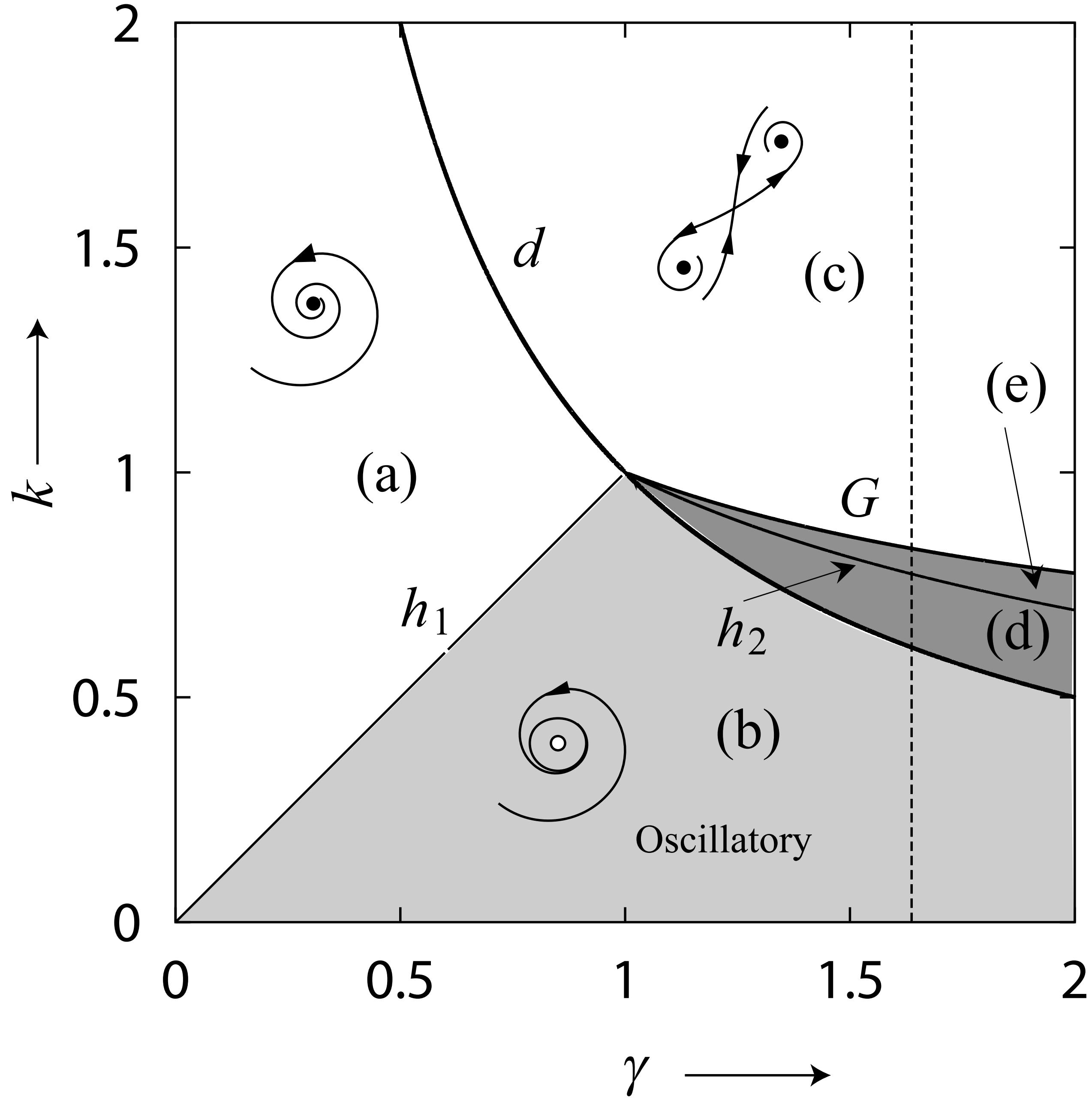
- ✎ **synchronization**
- ✎ **global/local bifurcations**

BVP oscillator

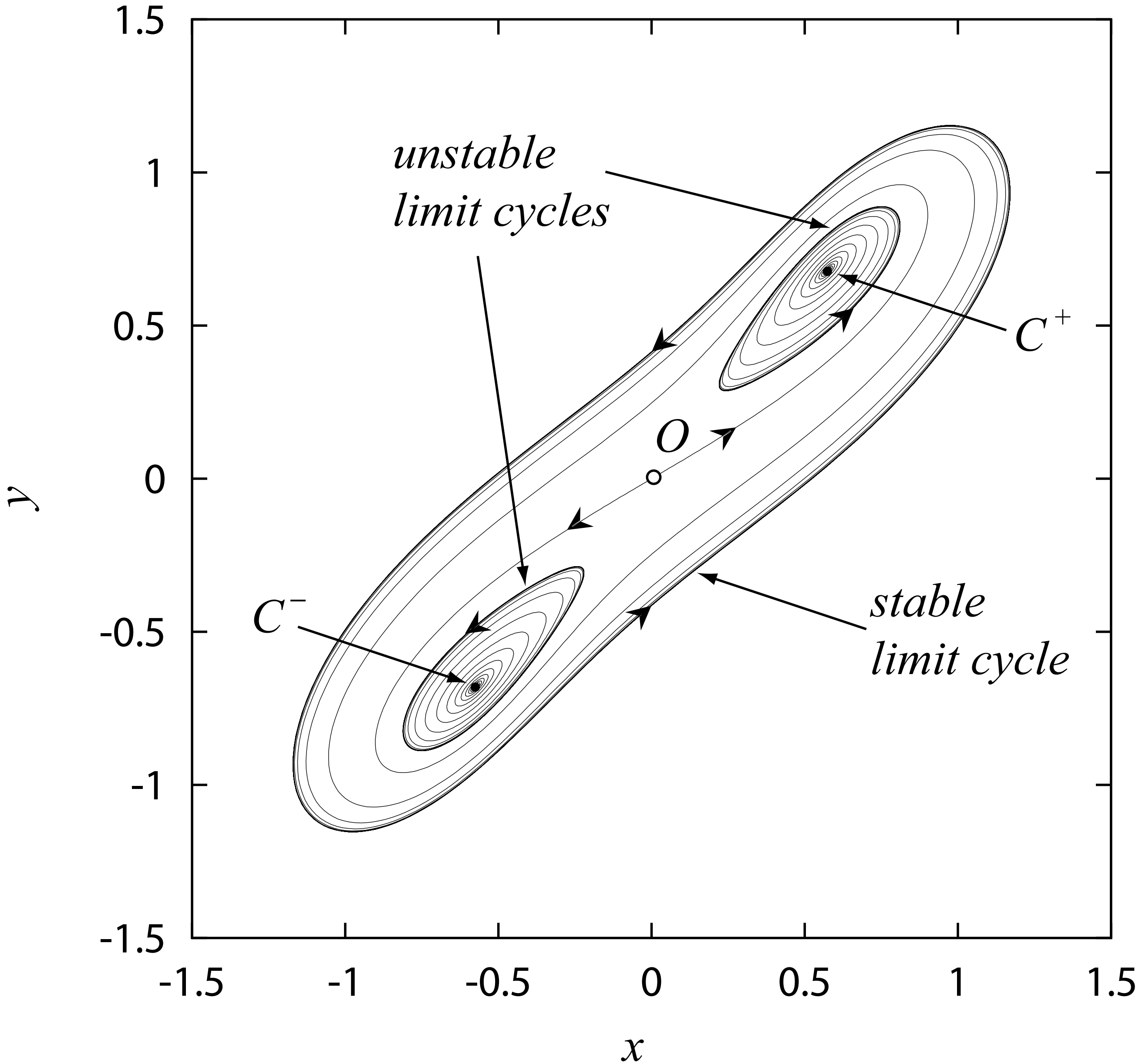


- 2 dimensional system
- 2 interface ports
- 2 essential parameters
- any mutual coupling model can be constructed
- Coupled by resisters ⇒ diffusive coupling

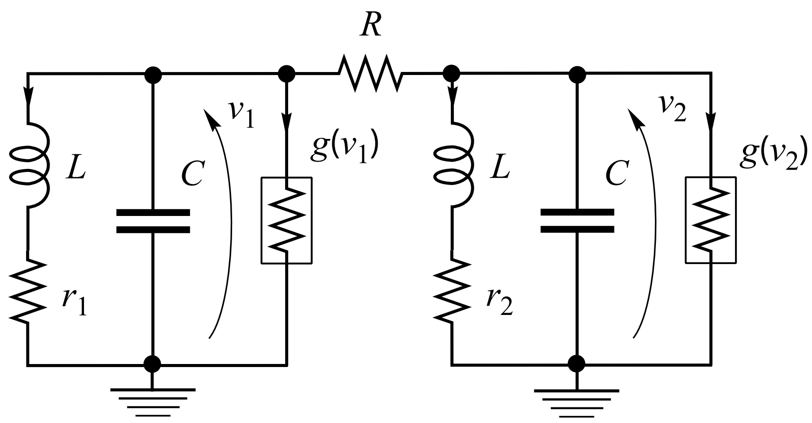
Bifurcations in a BVP oscillator



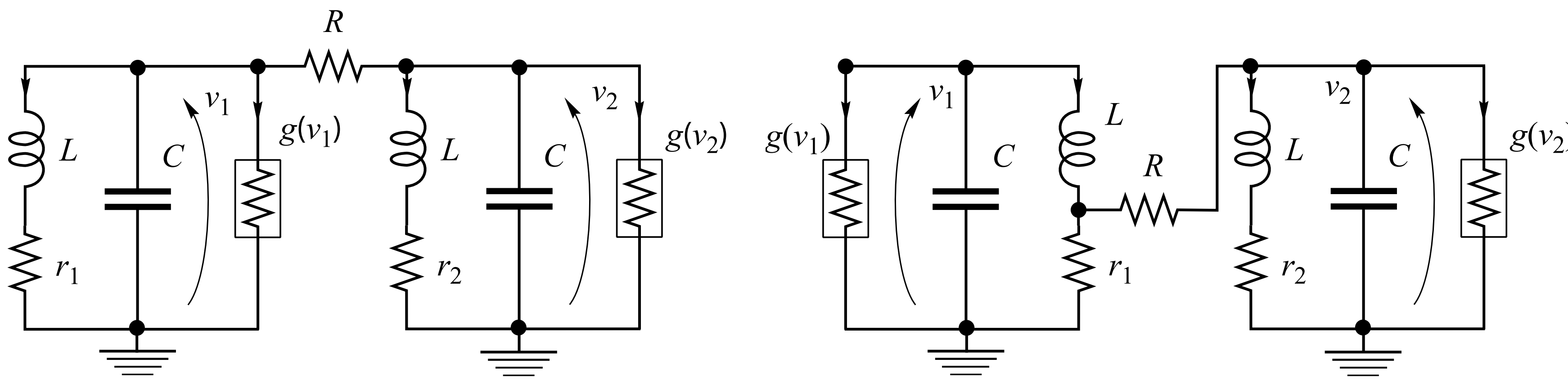
Bifurcations in a BVP oscillator



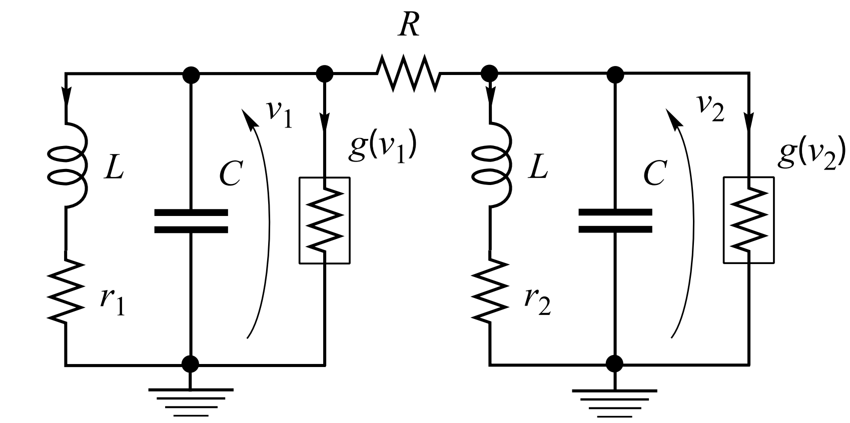
Resistively coupled BVP oscillators

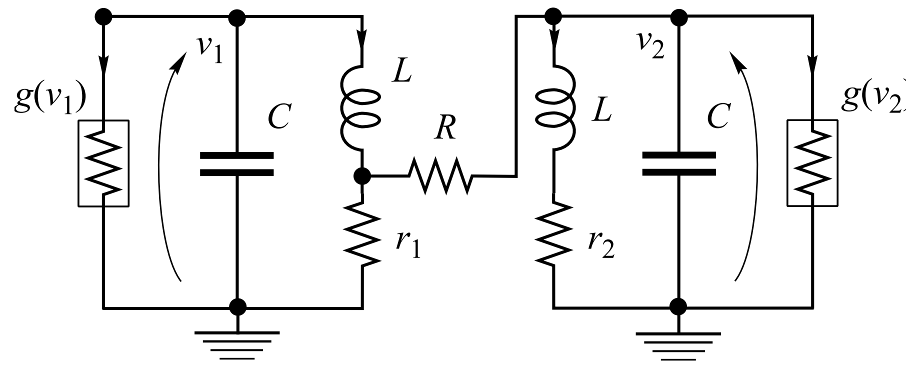


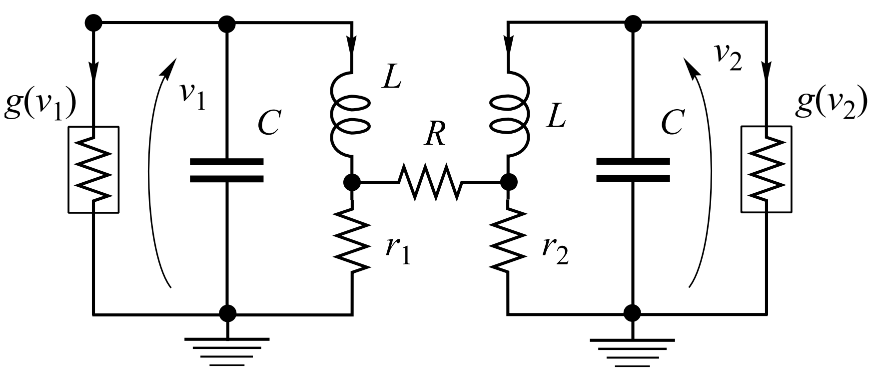
Resistively coupled BVP oscillators



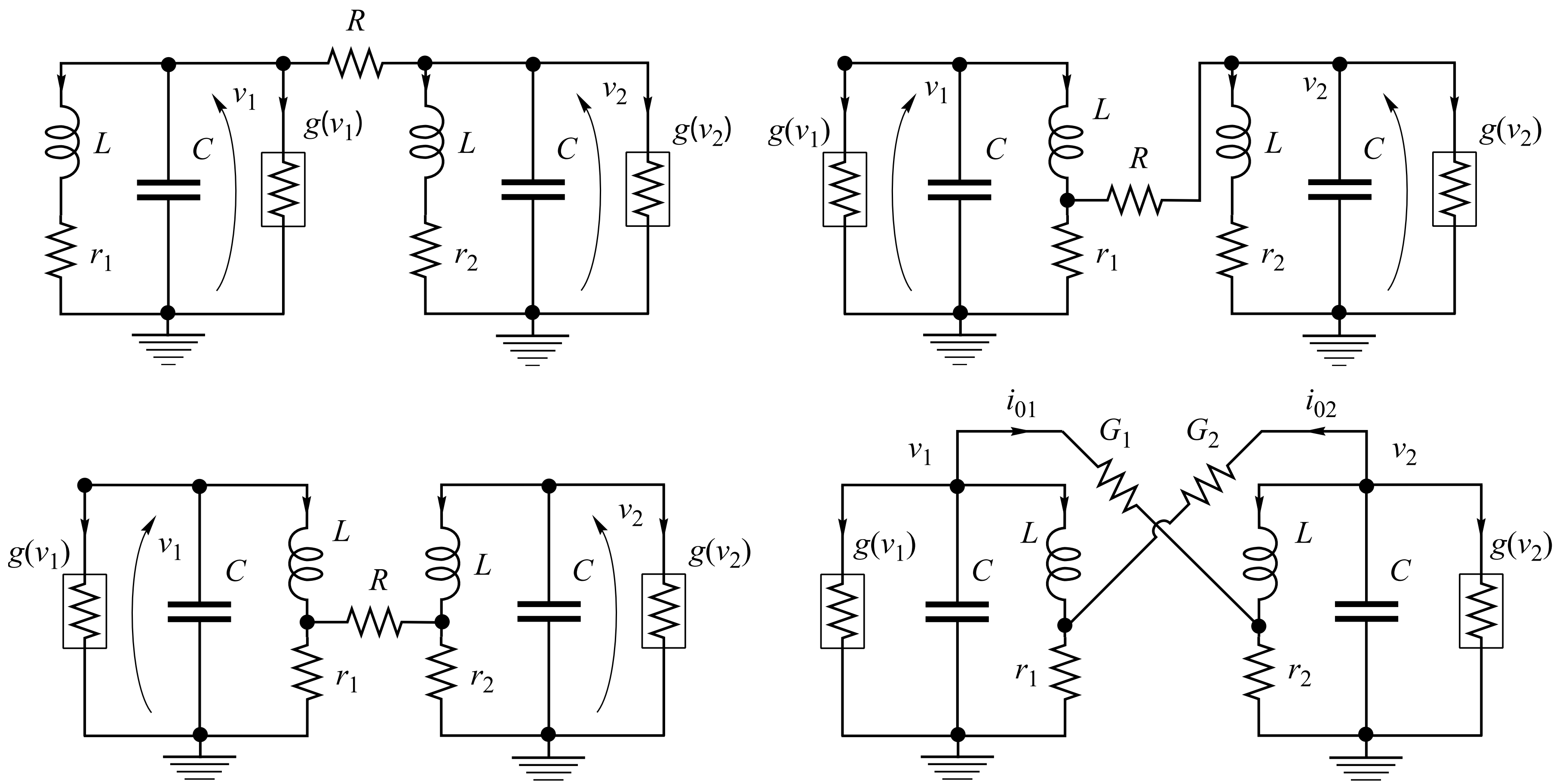
Resistively coupled BVP oscillators



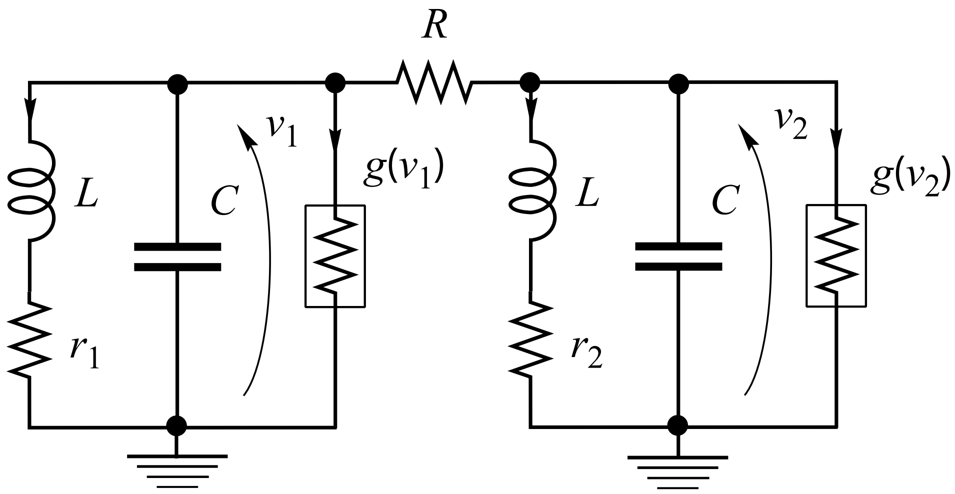


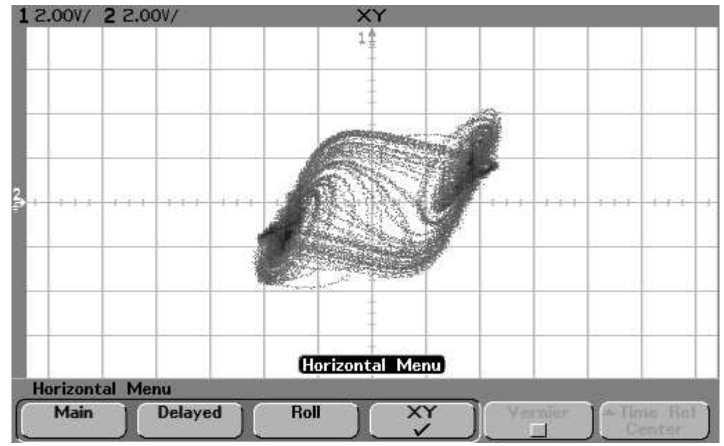


Resistively coupled BVP oscillators



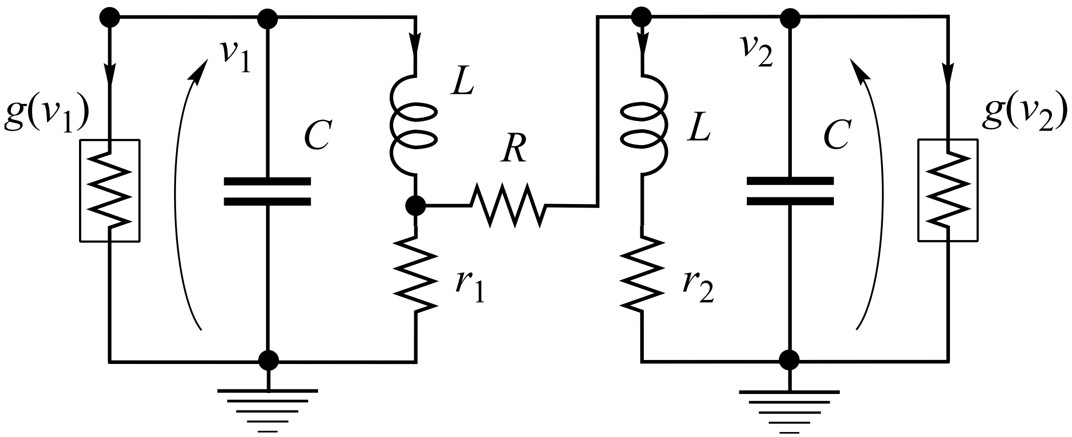
v-v coupled BVP oscillators

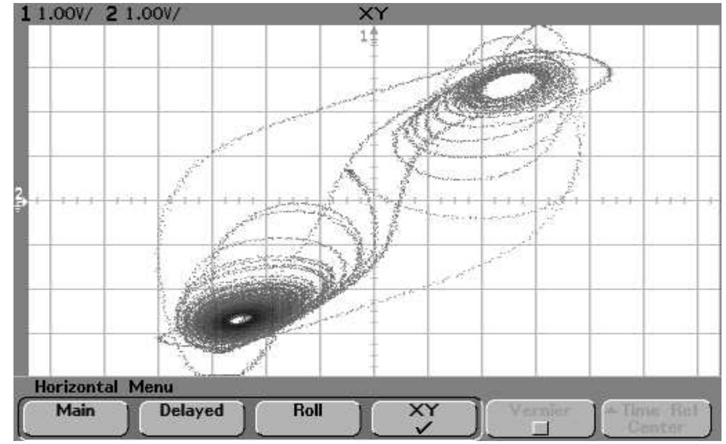




- ✎ **T. Ueta,** ***et al.*****, Strange attractor in resistively coupled BVP oscillators, In Proc. 2001 Int. Conf. on Progress in Nonlinear Science, Russia, July 2001, International Journal of Bifurcation and Chaos (to appear)**

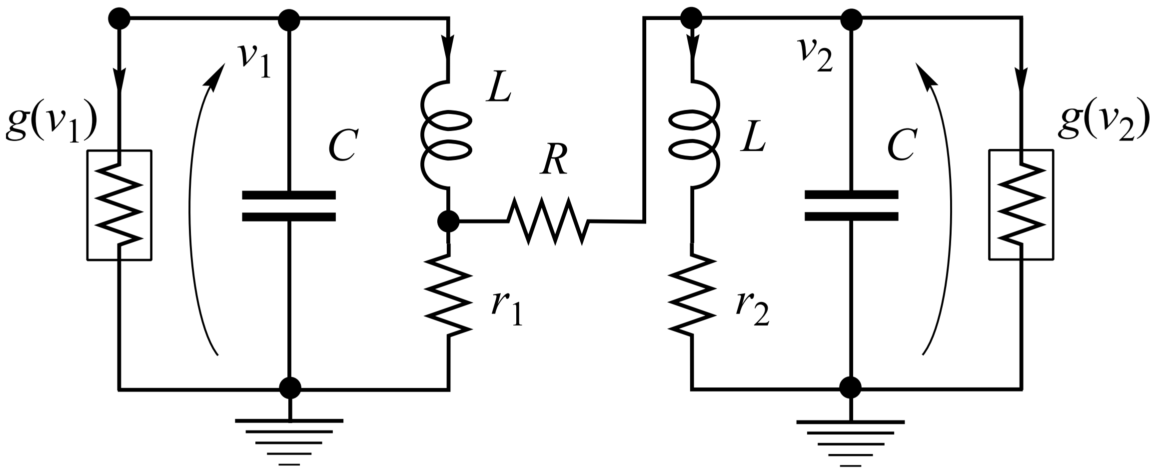
v-i coupled BVP oscillators



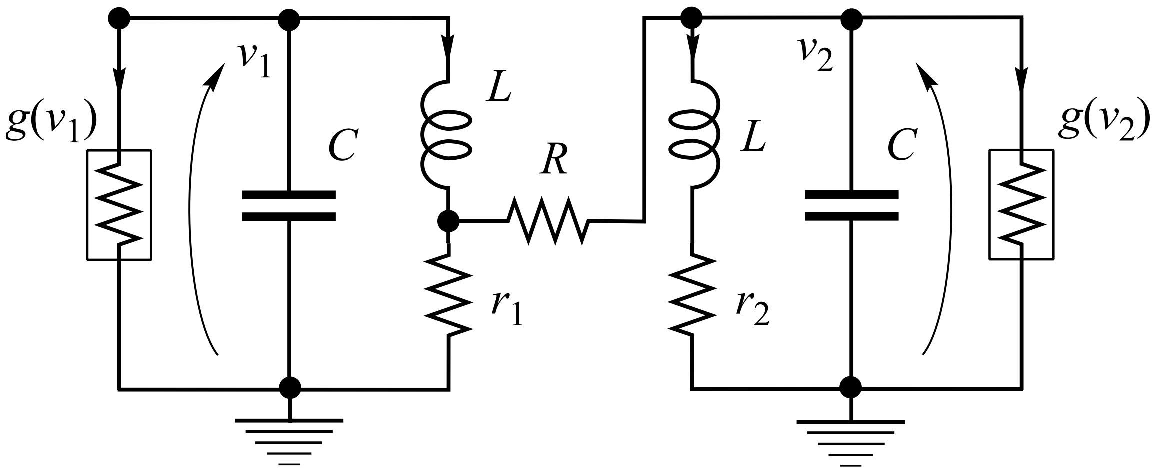


- ✎ **T. Ueta,** *et al.*, **Bifurcation and Chaos in Asymmetrically Coupled BVP Oscillators, ISCAS 2002, Scottsdale, Arizona, Int. J. Bifurcation and Chaos, Vol. 13, No. 5, 2003.**

Application. . .

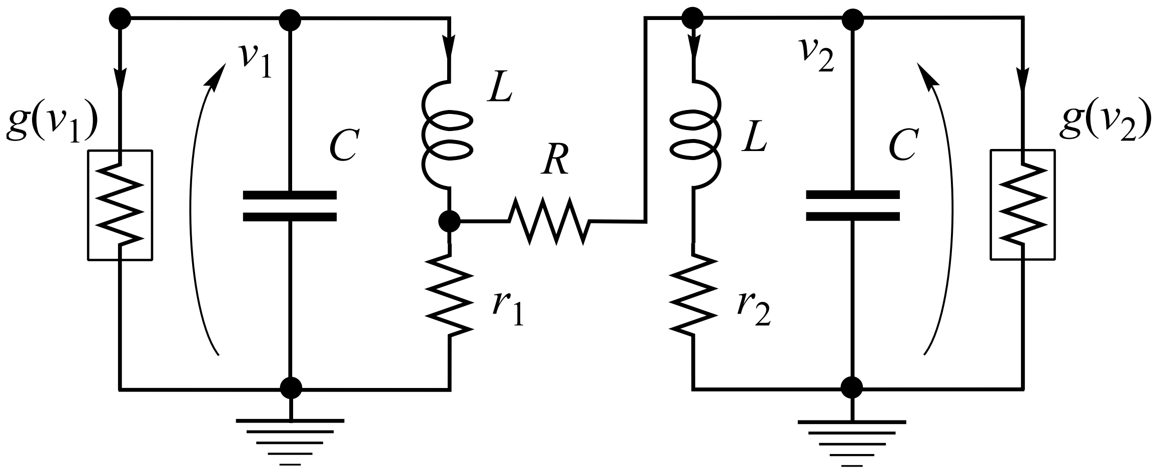


Application. . .



removing $g(v_2)$, letting $r_2 = \infty$, $R = 0$,

Application. . .



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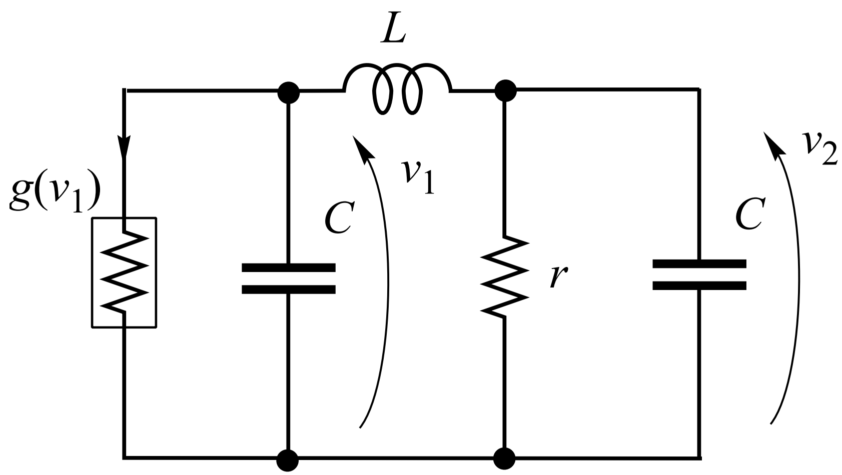
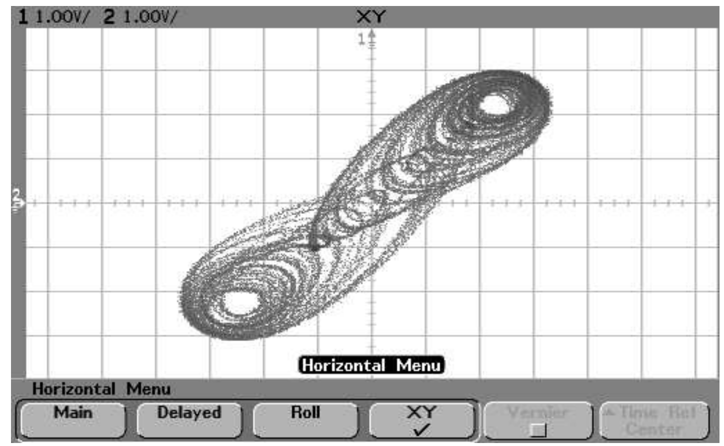
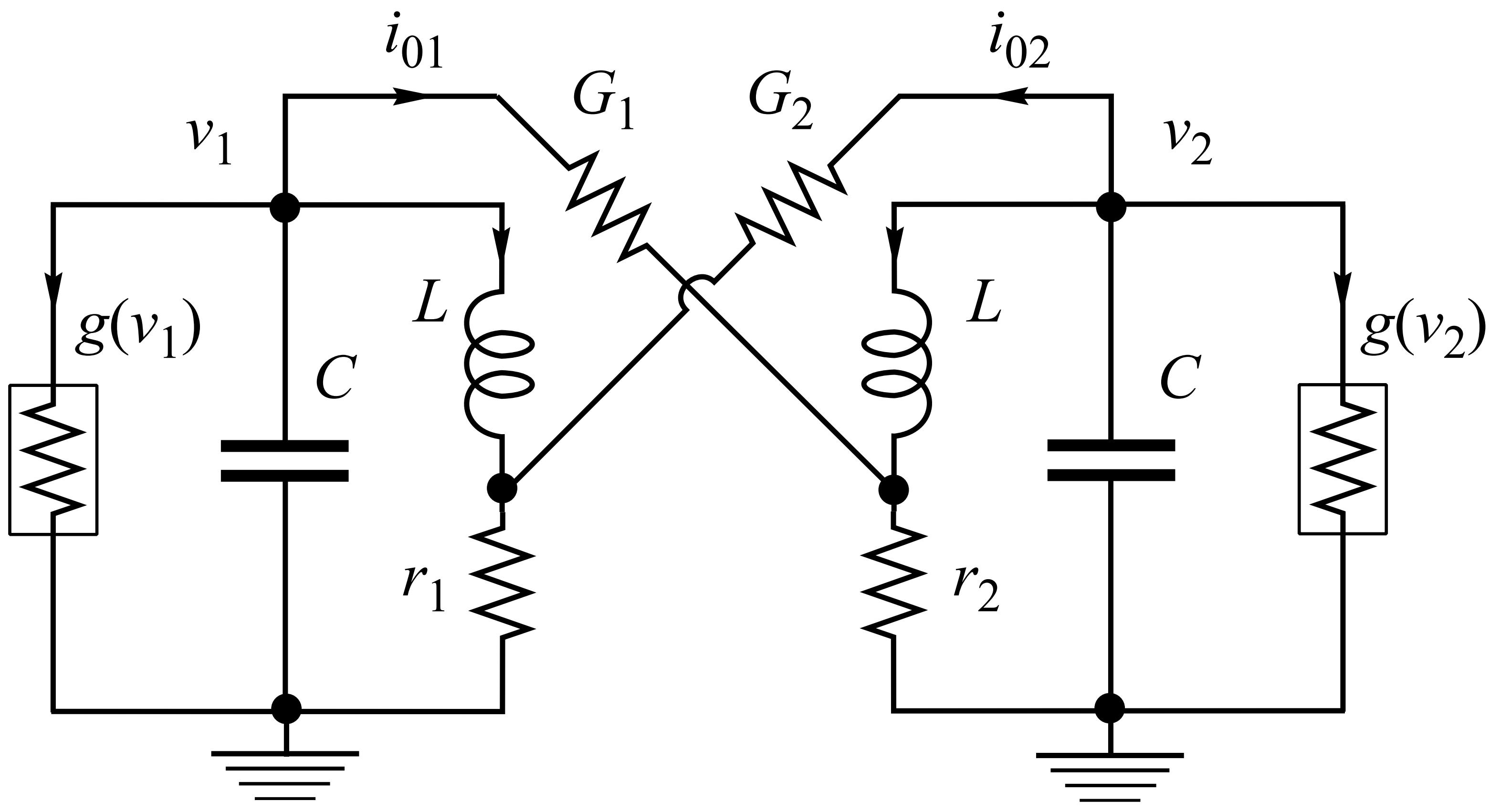




Cross-coupled BVP oscillators



Preceding researches

- ✎ **O. Papy and H. Kawakami, "Symmetrical Properties and Bifurcations of the Periodic Solutions for a Hybridly Coupled Oscillator," IEICE Trans., Vol. E78-A, No. 12, pp.1816–1821, 1995.**
- ✎ **O. Papy and H. Kawakami, "Symmetry Breaking and Recovering in a System of n Hybridly Coupled Oscillators," IEICE Trans. Vol. E79-A, No. 10, pp. 1581–1586, 1996.**

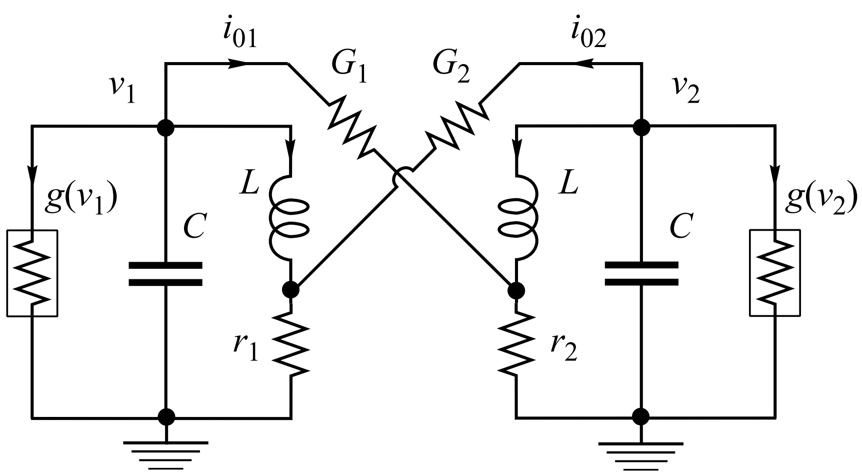
$\Rightarrow$ focused on only synchronization phenomena of limit cycles

In this talk...

Revisit hybridly coupled BVP oscillators in implementation point of view

- ✎ **Local bifurcations**
- ✎ **Torus doubling and breakdown**
- ✎ **Circuit implementation**
- ✎ **Chaos via period-doubling**

Circuit equation



$$
\begin{aligned}
C\frac{dv_1}{dt} &= -i_1 - g(v_1) - i_{01} \\
L\frac{di_1}{dt} &= v_1 - r_1(i_1 + i_{02}) \\
C\frac{dv_2}{dt} &= -i_2 - g(v_2) - i_{02} \\
L\frac{di_1}{dt} &= v_1 - r_1(i_1 + i_{01})
\end{aligned}
$$

$$
\begin{aligned}
i_{01} &= G_1\left(v_1 - r_2(i_2 + i_{01})\right) \\
i_{02} &= G_2\left(v_2 - r_1(i_1 + i_{02})\right).
\end{aligned}
$$

$$
\begin{aligned}
C\frac{dv_1}{dt} &= -i_1 - g(v_1) - \frac{G_1}{1+G_1 r_2}(v_1 - r_2 i_2) \\
L\frac{di_1}{dt} &= v_1 - r_1 i_1 - \frac{G_2 r_1}{1+G_2 r_1}(v_2 - r_1 i_1) \\
C\frac{dv_2}{dt} &= -i_2 - g(v_2) - \frac{G_2}{1+G_2 r_1}(v_2 - r_1 i_1) \\
L\frac{di_2}{dt} &= v_2 - r_2 i_2 - \frac{G_1 r_2}{1+G_1 r_2}(v_1 - r_2 i_2)
\end{aligned}
$$

Variable transformation

$$x_j = \frac{v_j}{a}\sqrt{\frac{C}{L}}, \quad y_j = \frac{i_j}{a}, \quad k_j = r_j\sqrt{\frac{C}{L}},$$

$$\delta_j = G_j\sqrt{\frac{L}{C}}, \quad j = 1, 2.$$

$$\tau = \frac{1}{\sqrt{LC}}t, \quad \gamma = ab\sqrt{\frac{L}{C}},$$

A nonlinear negative conductance

$$g(v) = -a\tanh bv$$

Normalized ODE

$$\begin{cases} \dot{x}_1 &= -y_1 + \tanh \gamma x_1 - \dfrac{\delta_1}{1+\delta_1 k_2}(x_1 - k_2 y_2) \\ \dot{y}_1 &= x_1 - k_1 y_1 - \dfrac{\delta_2 k_1}{1+\delta_2 k_1}(x_2 - k_1 y_1) \\ \dot{x}_2 &= -y_2 + \tanh \gamma x_2 - \dfrac{\delta_2}{1+\delta_2 k_1}(x_2 - k_1 y_1) \\ \dot{y}_2 &= x_2 - k_2 y_2 - \dfrac{\delta_1 k_2}{1+\delta_1 k_2}(x_1 - k_2 y_2) \end{cases}$$

x**: voltage,** y**: current**

Symmetry

- **Symmetry in the state space:**

$$\begin{aligned} P : \boldsymbol{R}^4 &\rightarrow \boldsymbol{R}^4 \\ (x_1, y_1, x_2, y_2) &\mapsto (-x_1, -y_1, -x_2, -y_2). \end{aligned}$$

  The ODE is invariant under this transformation.

- **Symmetry in the parameter space:**

$$(k_1, k_2, \delta_1, \delta_2) \rightarrow (k_2, k_1, \delta_2, \delta_1)$$

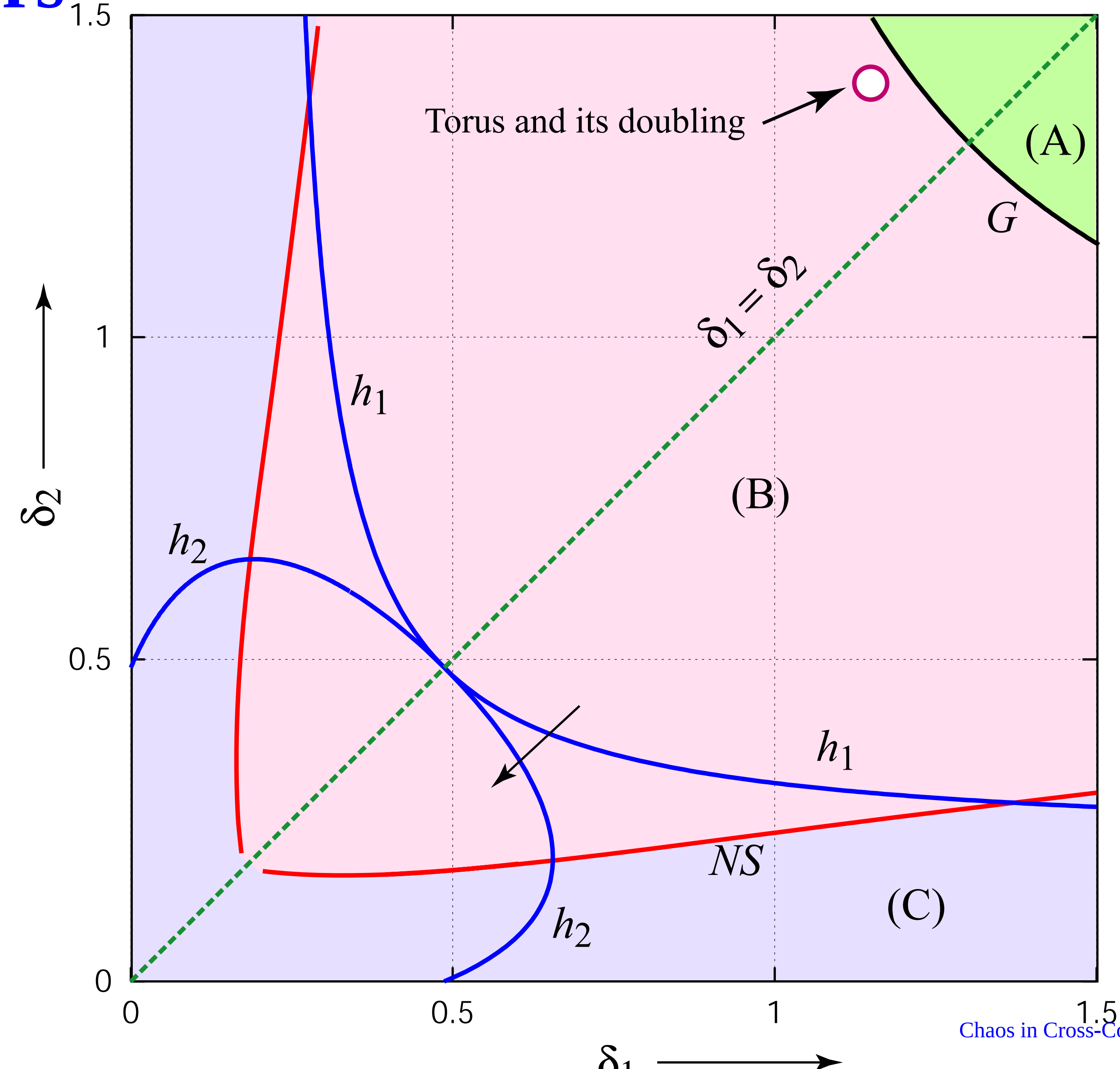
Fixing parameters

$$\begin{cases} \dot{x}_1 & = & -y_1 + \tanh \gamma x_1 - \dfrac{\delta_1}{1+\delta_1 k_2}(x_1 - k_2 y_2) \\ \dot{y}_1 & = & x_1 - k_1 y_1 - \dfrac{\delta_2 k_1}{1+\delta_2 k_1}(x_2 - k_1 y_1) \\ \dot{x}_2 & = & -y_2 + \tanh \gamma x_2 - \dfrac{\delta_2}{1+\delta_2 k_1}(x_2 - k_1 y_1) \\ \dot{y}_2 & = & x_2 - k_2 y_2 - \dfrac{\delta_1 k_2}{1+\delta_1 k_2}(x_1 - k_2 y_2) \end{cases}$$

$\gamma \approx 1.6$

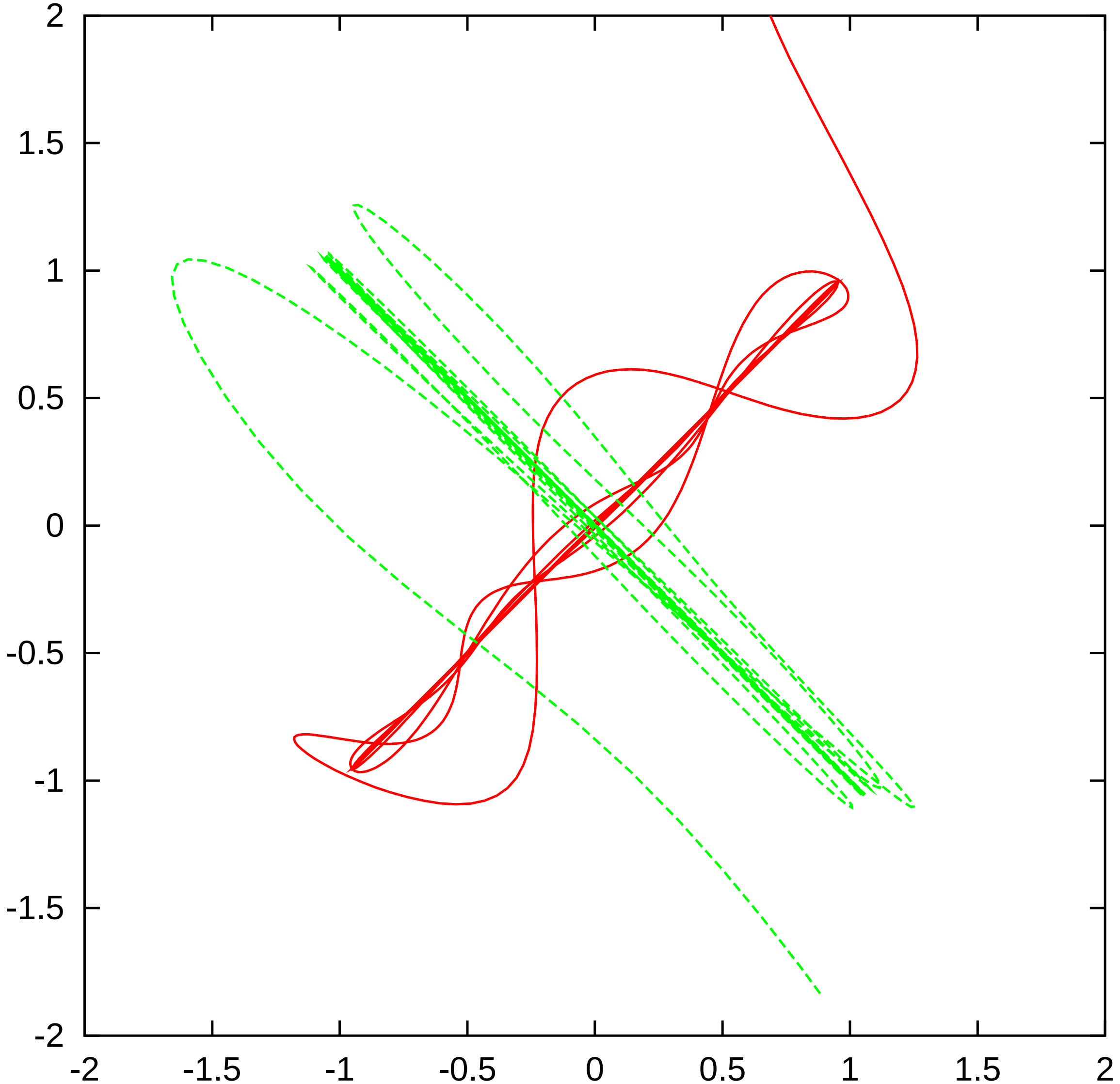
- ✎ $k_1 = k_2 = 0.75$**: identical oscillators case**
- ✎ **free parameters case**

Bifurcation in coupled identical oscillators

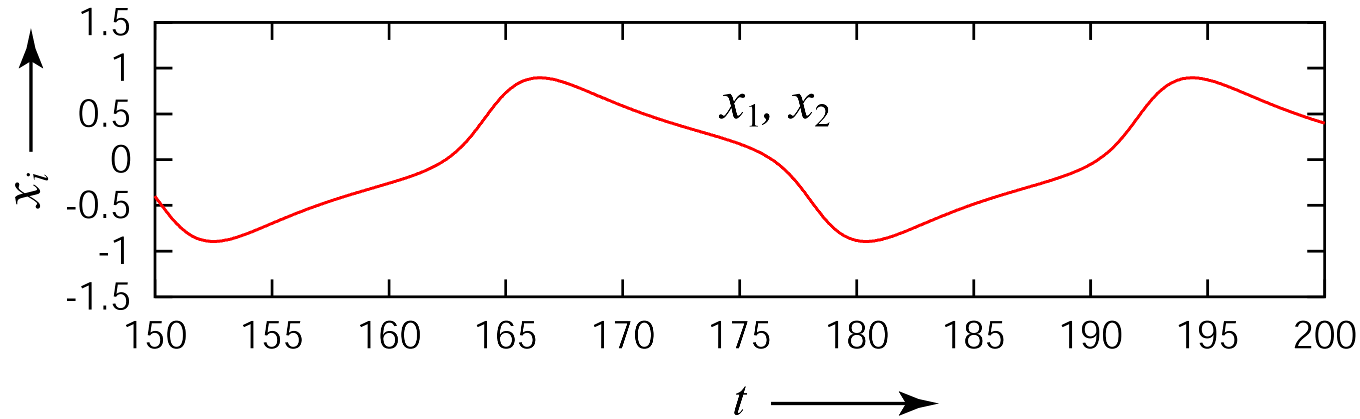


$$\delta_1 = \delta_2$$

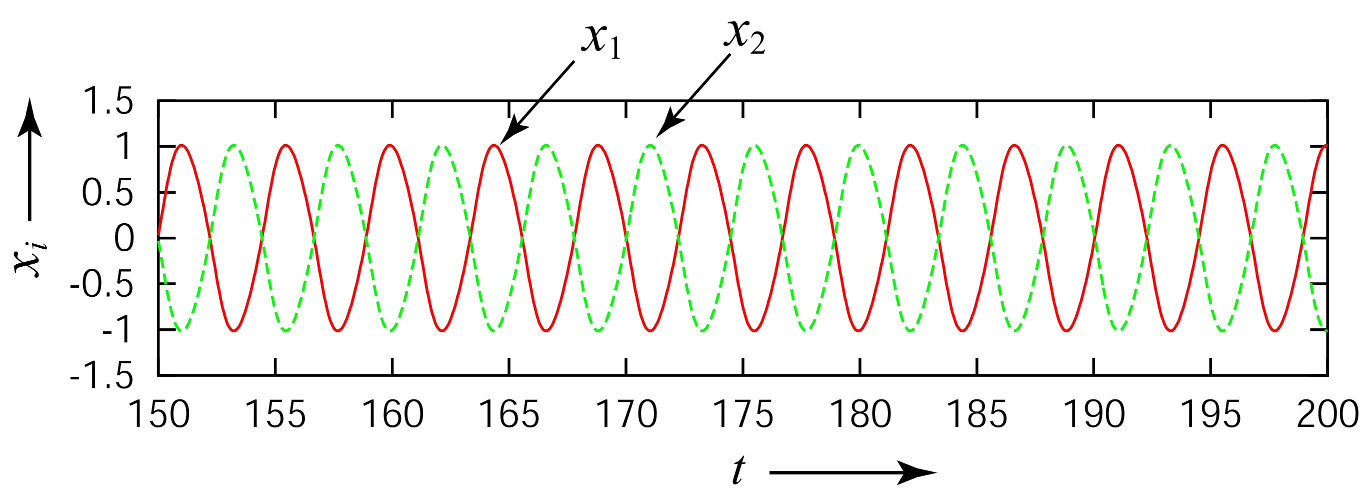
Phase portrait with different initial values in x_1-x_2 plane.



Time response



(a) completely in-phase synchronization



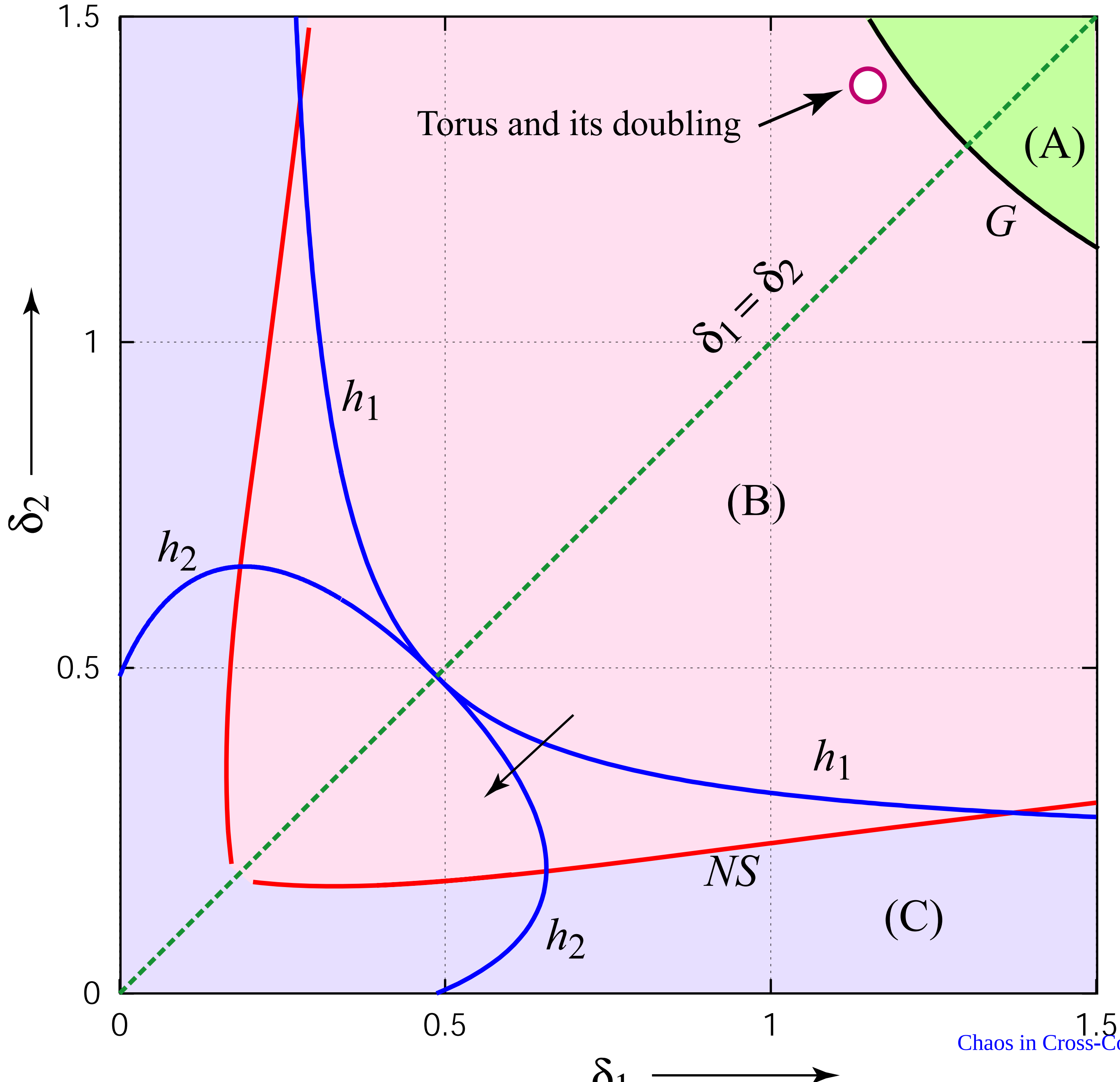
(b) completely anti-phase synchronization.

$\delta_1 = \delta_2 = 1.0$

Limit sets

- **In area (A) and almost all (B): There exist stable sinks C^{+} and C^{-}**
- **in-phase mode solution is disappeared by the tangent bifurcation.**
- **anti-phase mode solution is disappeared by the Neimark-Sacker bifurcaiton**

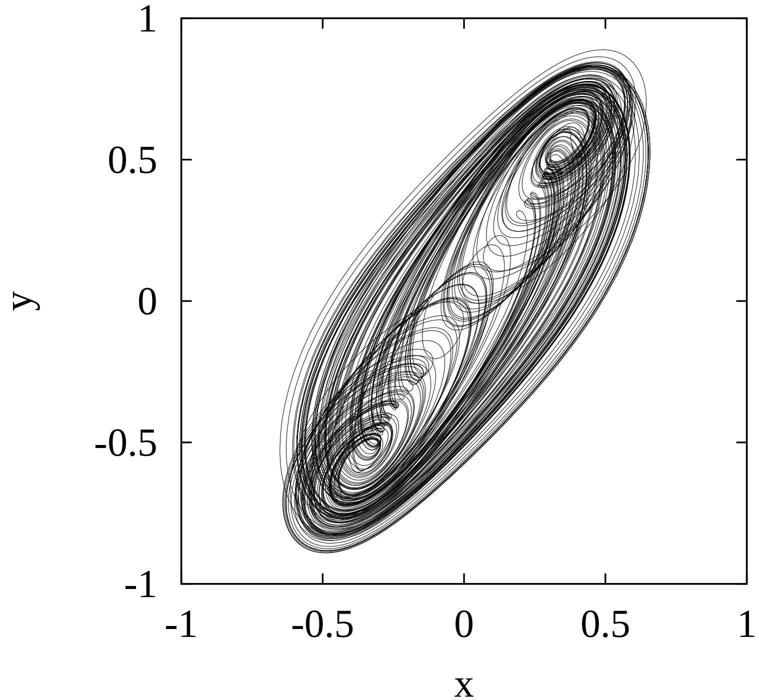
Bifurcation diagram



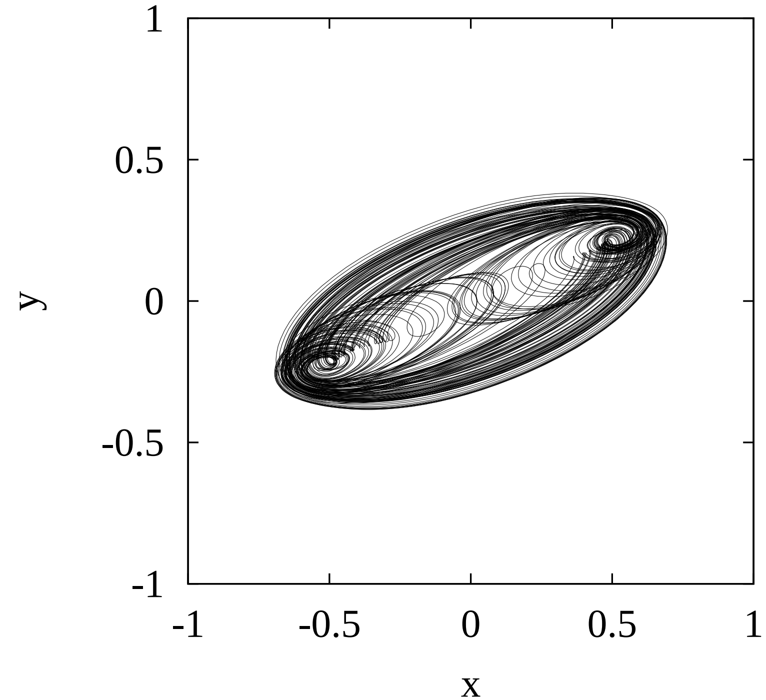
Classification

δ	**mode**
small (weak coupling)	**in-phase**
medium (moderate coupling)	**in-phase and anti-phase**
large (strong coupling)	**anti-phase**

Torus: $k_1 = k_2 = 0.75$, $\delta_1 = 1.4$, $\delta_2 = 1.2$.



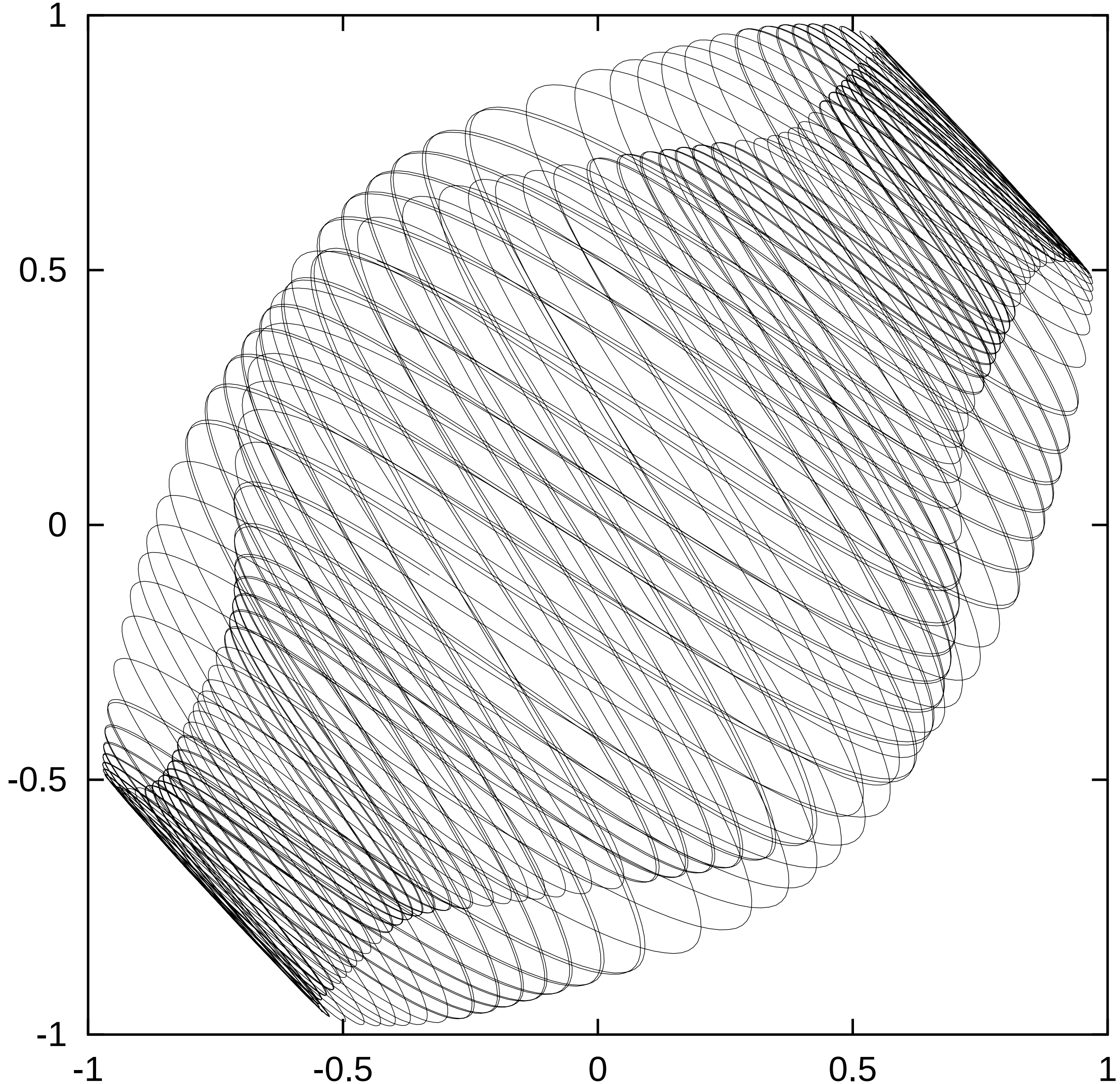
x_1-y_1 **plane**



x_2-y_2 **plane**

As a result of compromise between in-phase an anti-phase mode

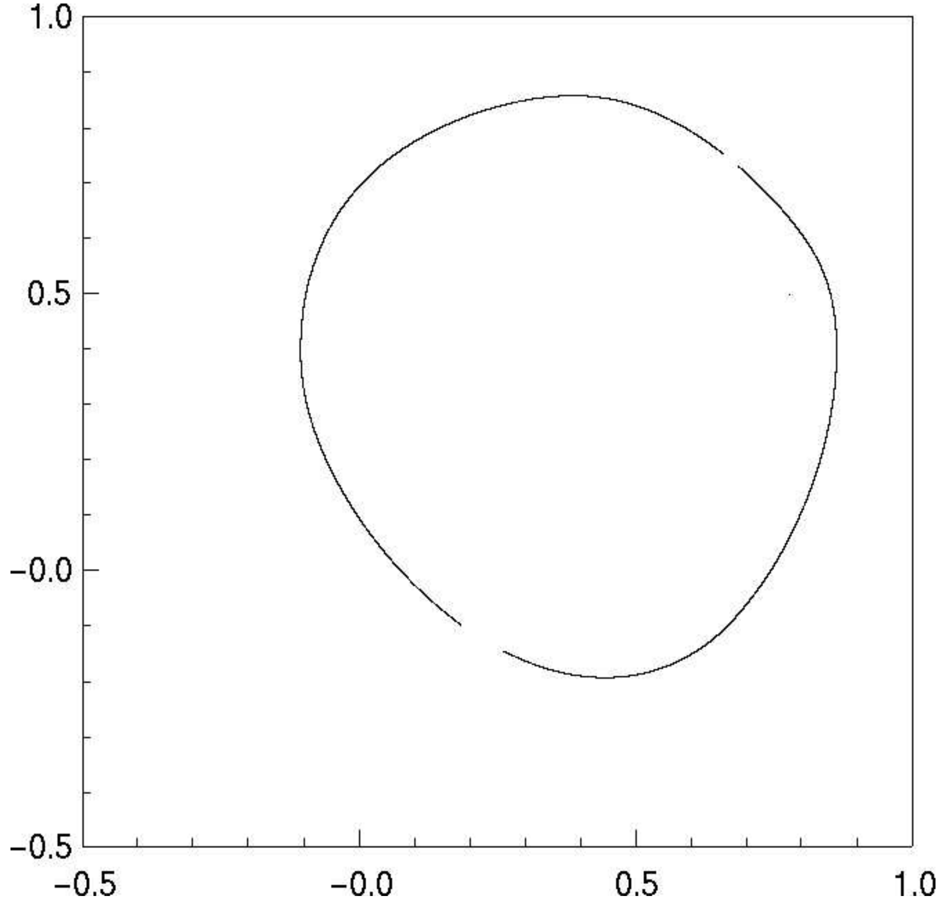
Torus: $k_1 = k_2 = 0.75$, $\delta_1 = 1.4$, $\delta_2 = 1.2$.



x_1-x_2 **plane**

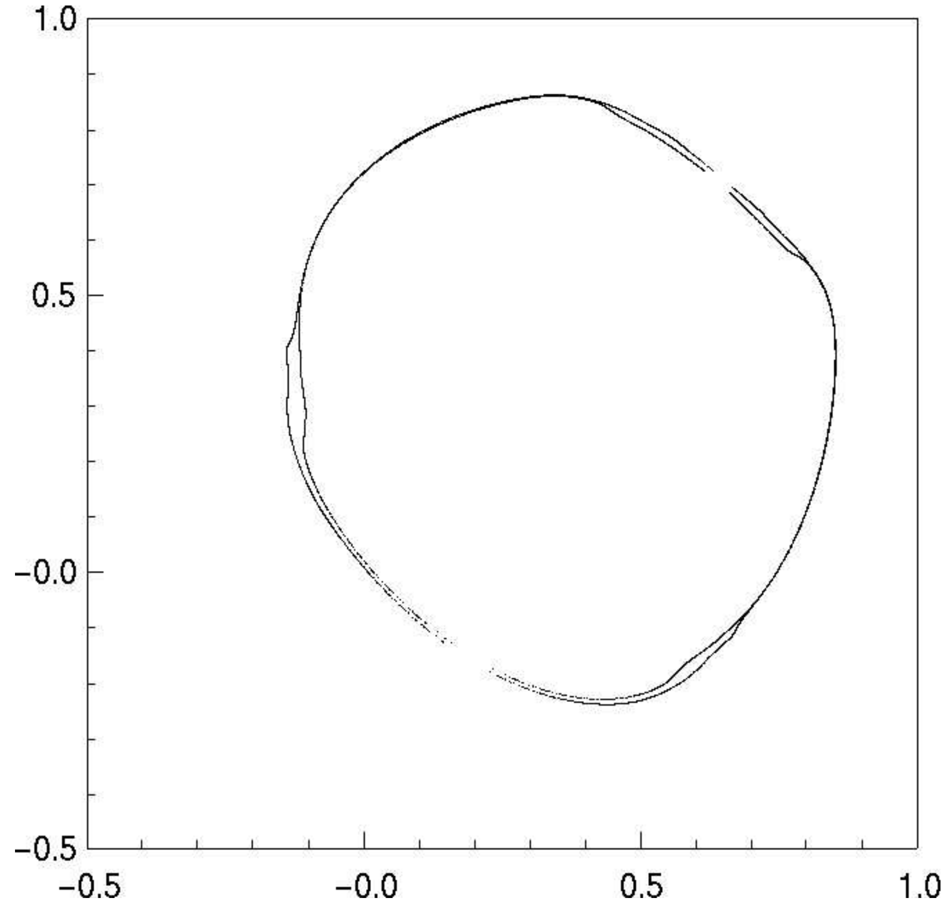
Poincaré mapping on $x_0 = 0.9$

y_1-y_2 **plane**



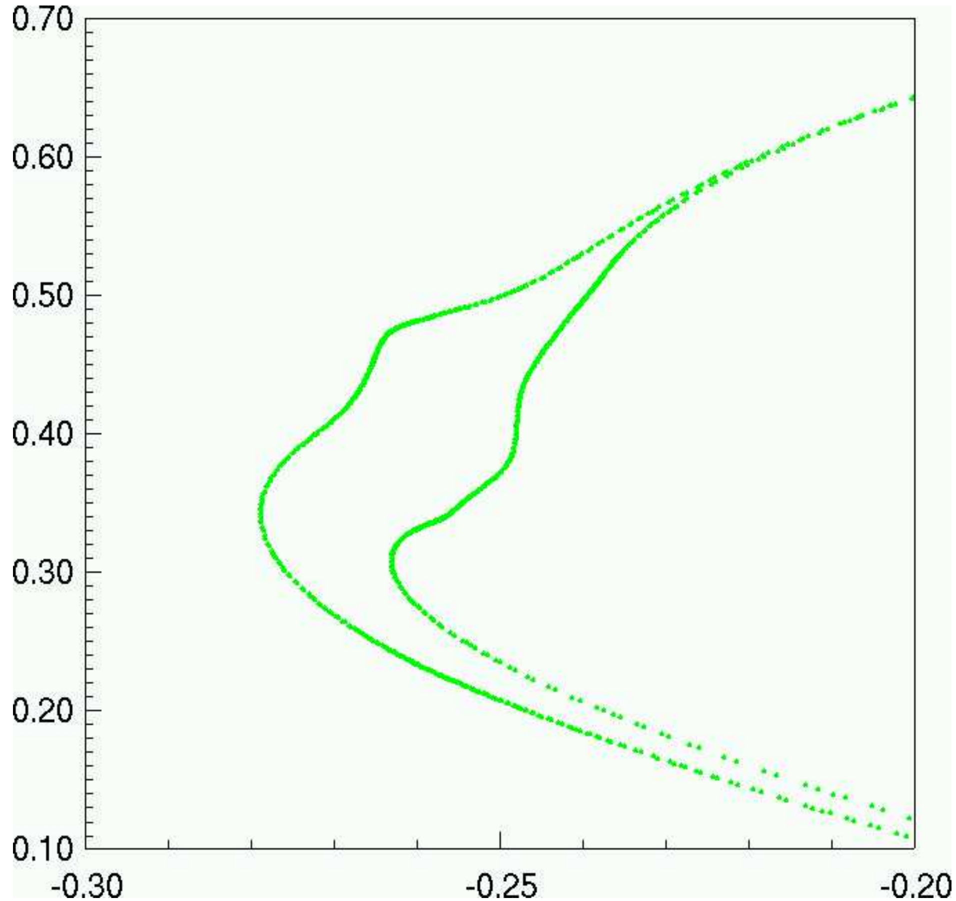
Torus doubling, $k_1 = k_2 = 0.75$**.** $\delta = 1.4989$**.**

y_1-y_2 **plane**



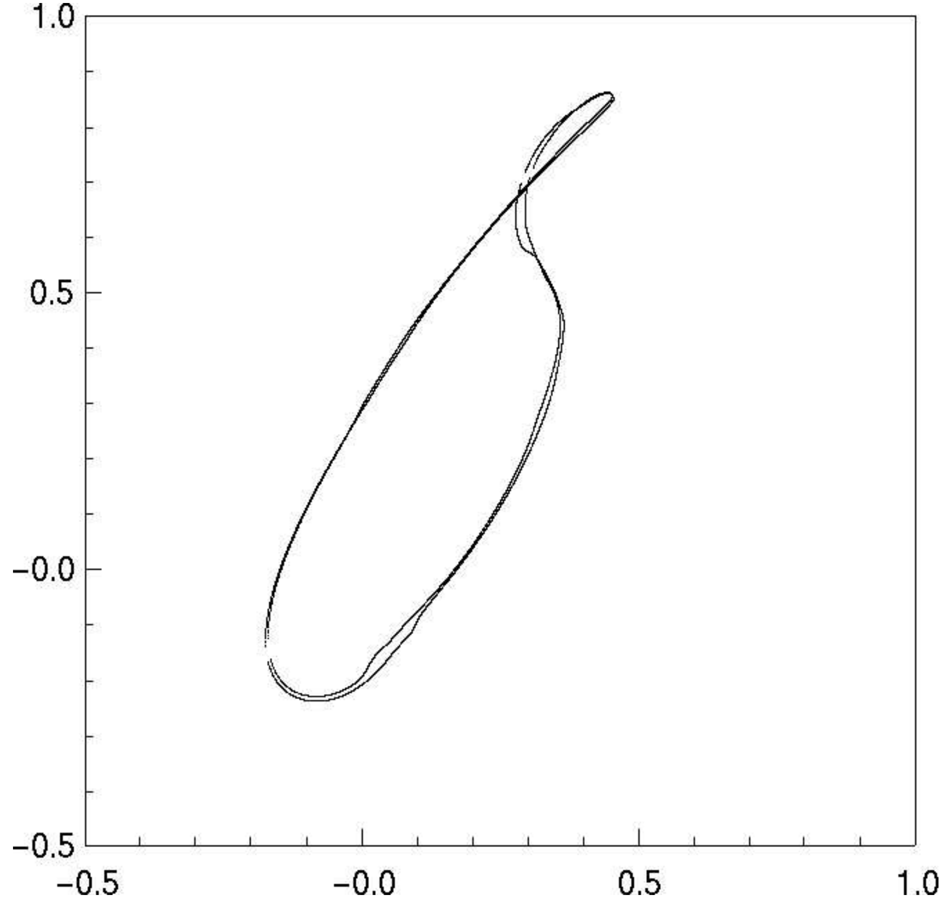
Torus doubling, $k_1 = k_2 = 0.75$**.** $\delta = 1.4989$**.**

y_1**-**y_2 **plane**



Poincaré mapping on $x_0 = 0.9$, $\delta = 1.4989$.

x_2-y_2 plane



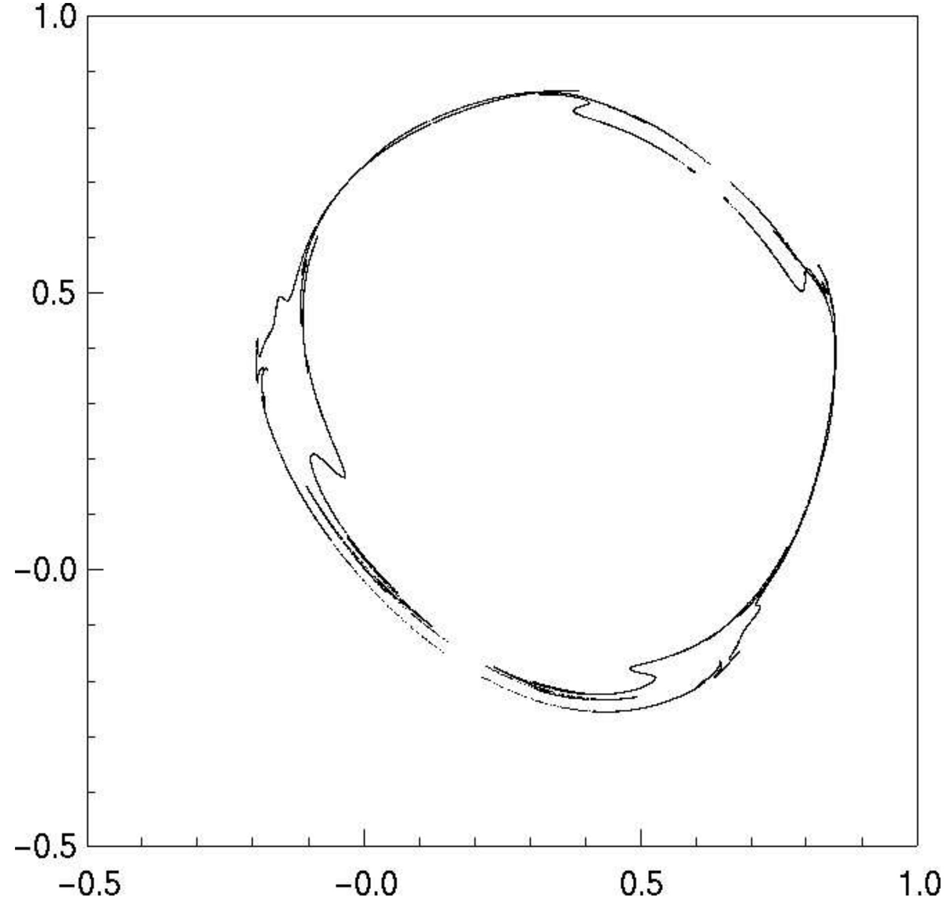
Torus doubling

M. Sekikawa, T. Miyoshi, and N. Inaba. "Successive Torus Doubling," IEEE Trans. Circuits and Systems-I, Vol. 48, No. 1, pp. 28–34, Jan 2001.

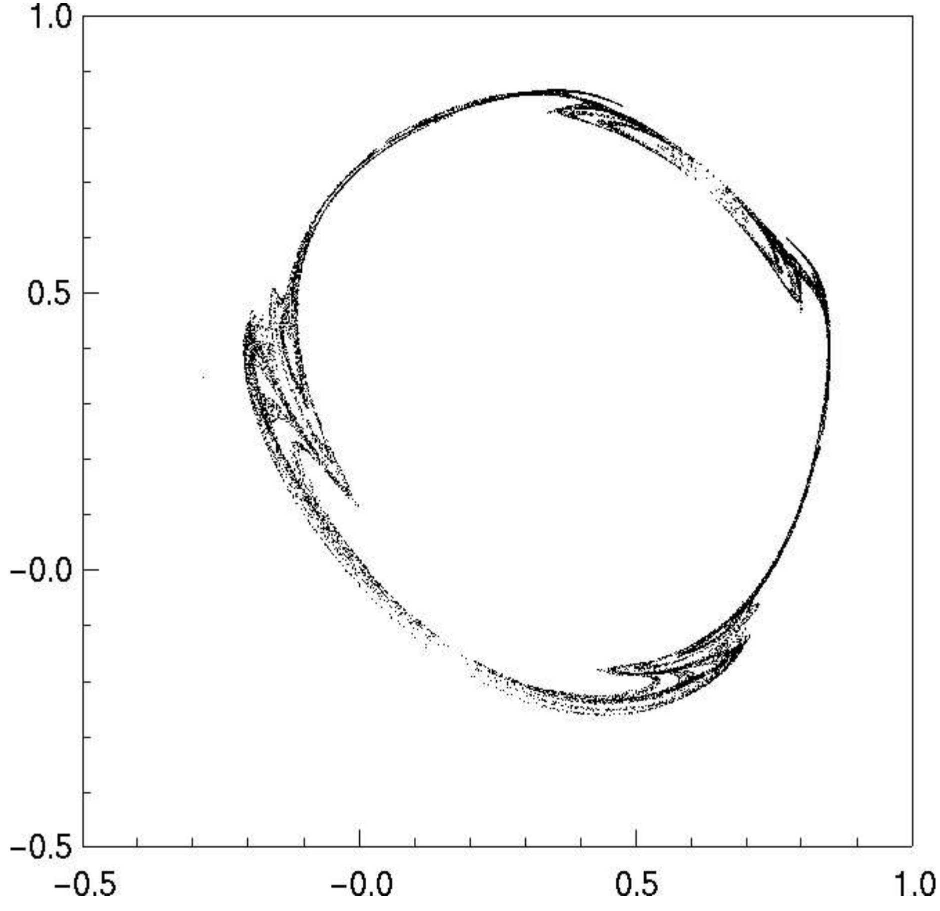
Cf. an attractor via "swollen bifurcation": $\Rightarrow$ no successive doubling is occurred.

$\Rightarrow$ New type of torus doubling ?

Chaos, $\delta = 1.522$

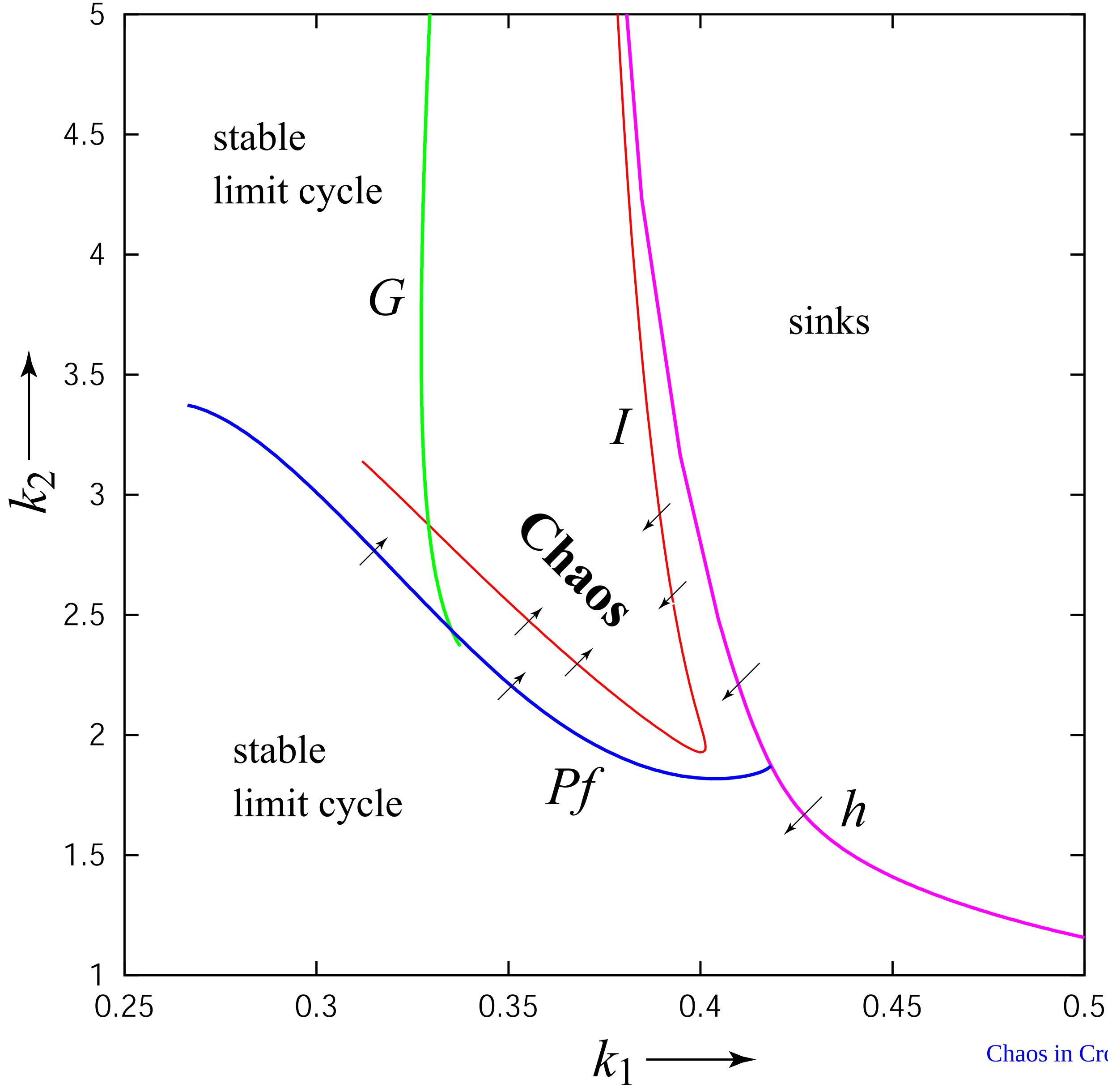


Chaos, $\delta = 1.535$

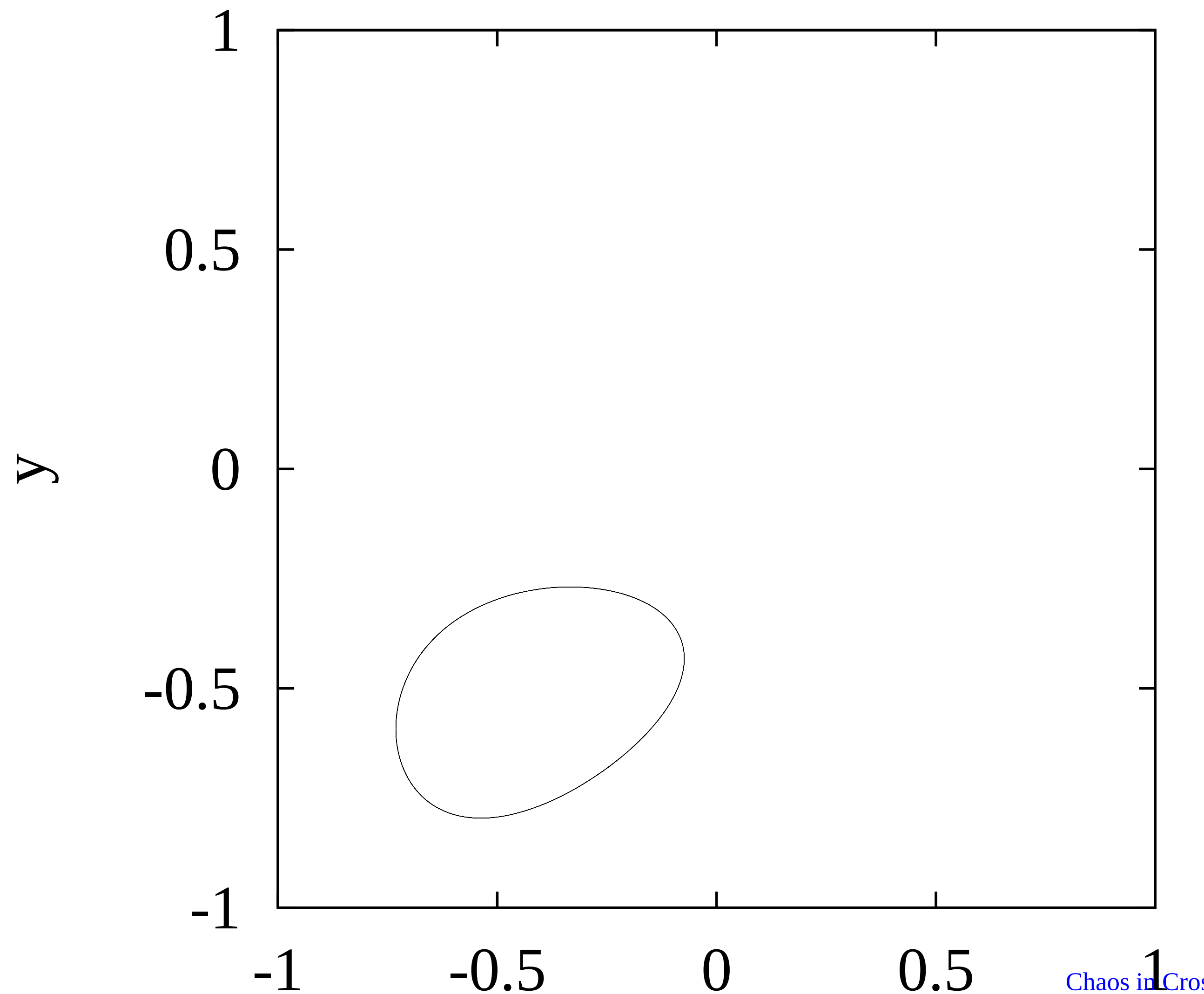


Asymmetrical case

$\delta_1 = 0.377$, $\delta_2 = 2.696$.



Phase portrait, $k_2 = 2.96$**,** $\delta_1 = 0.337$**,** $\delta_2 = 2.696$**.**



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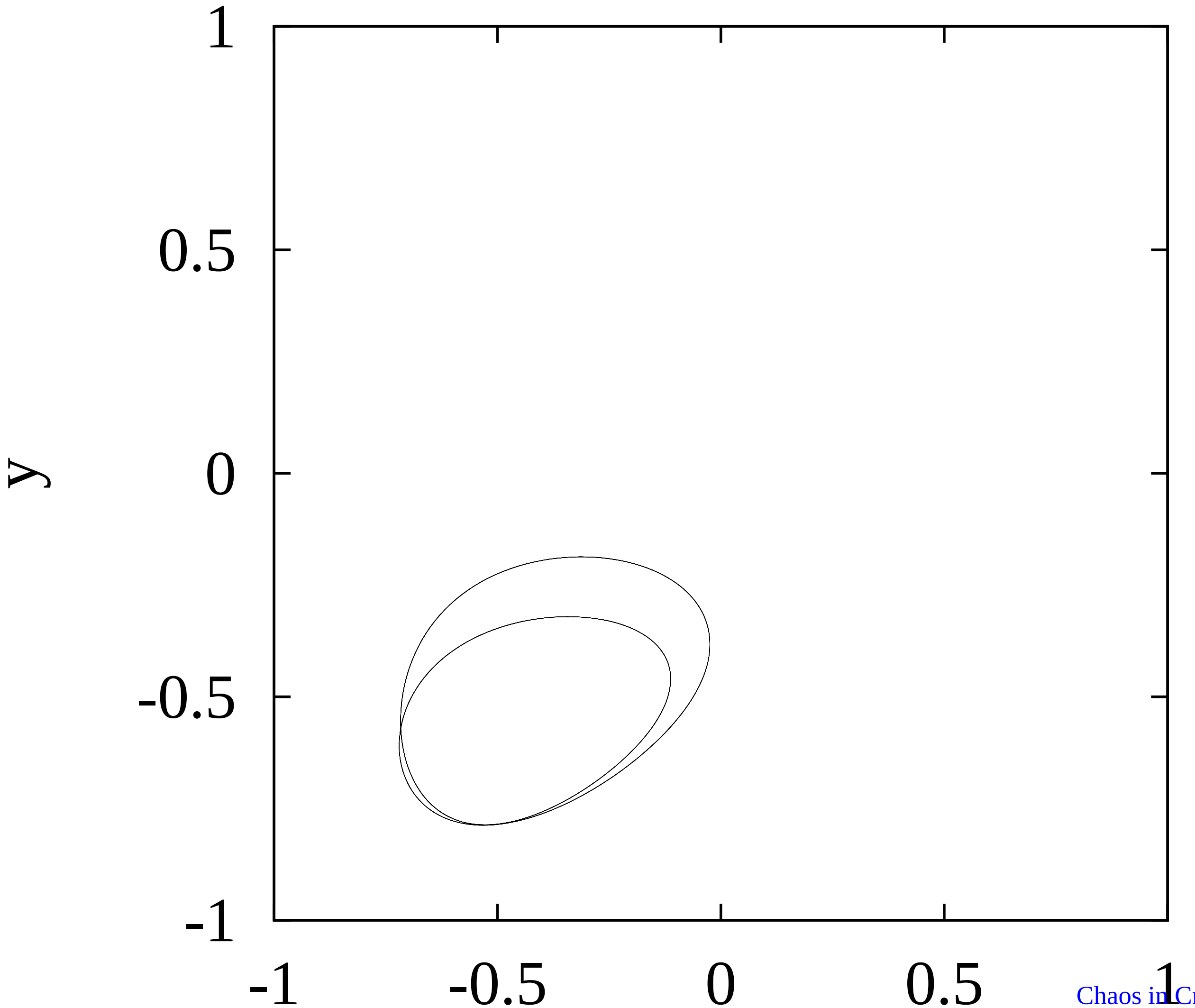

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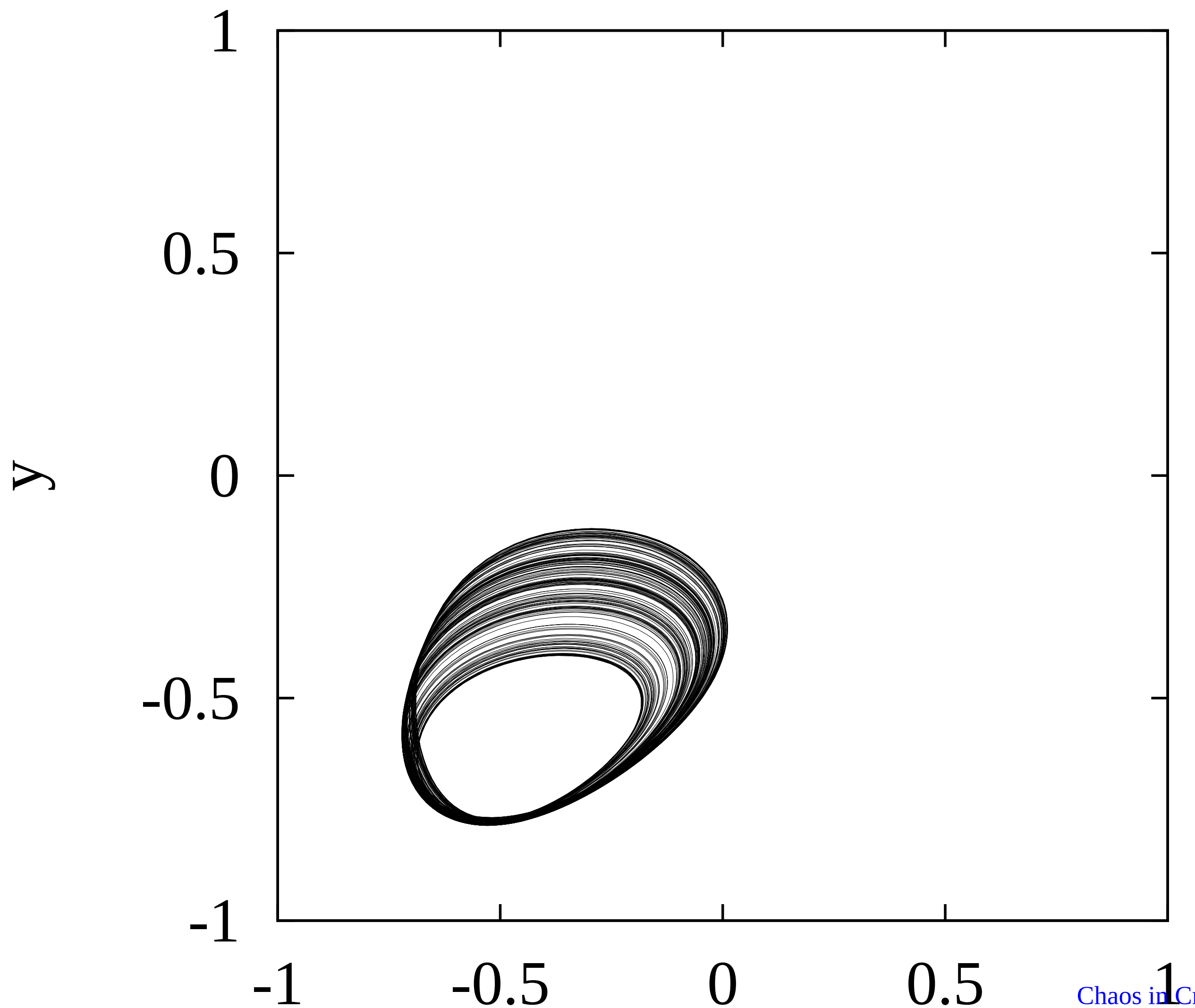

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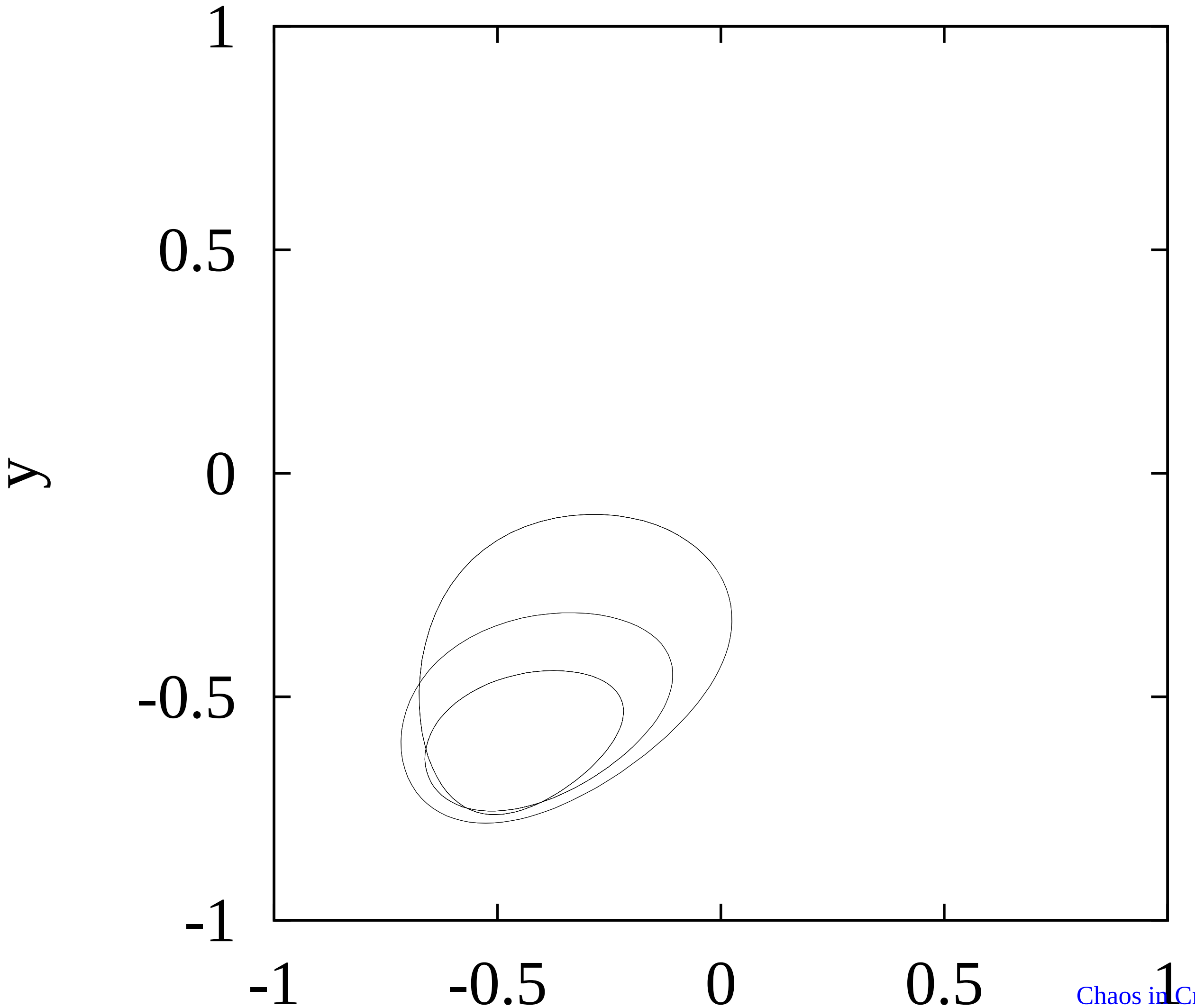

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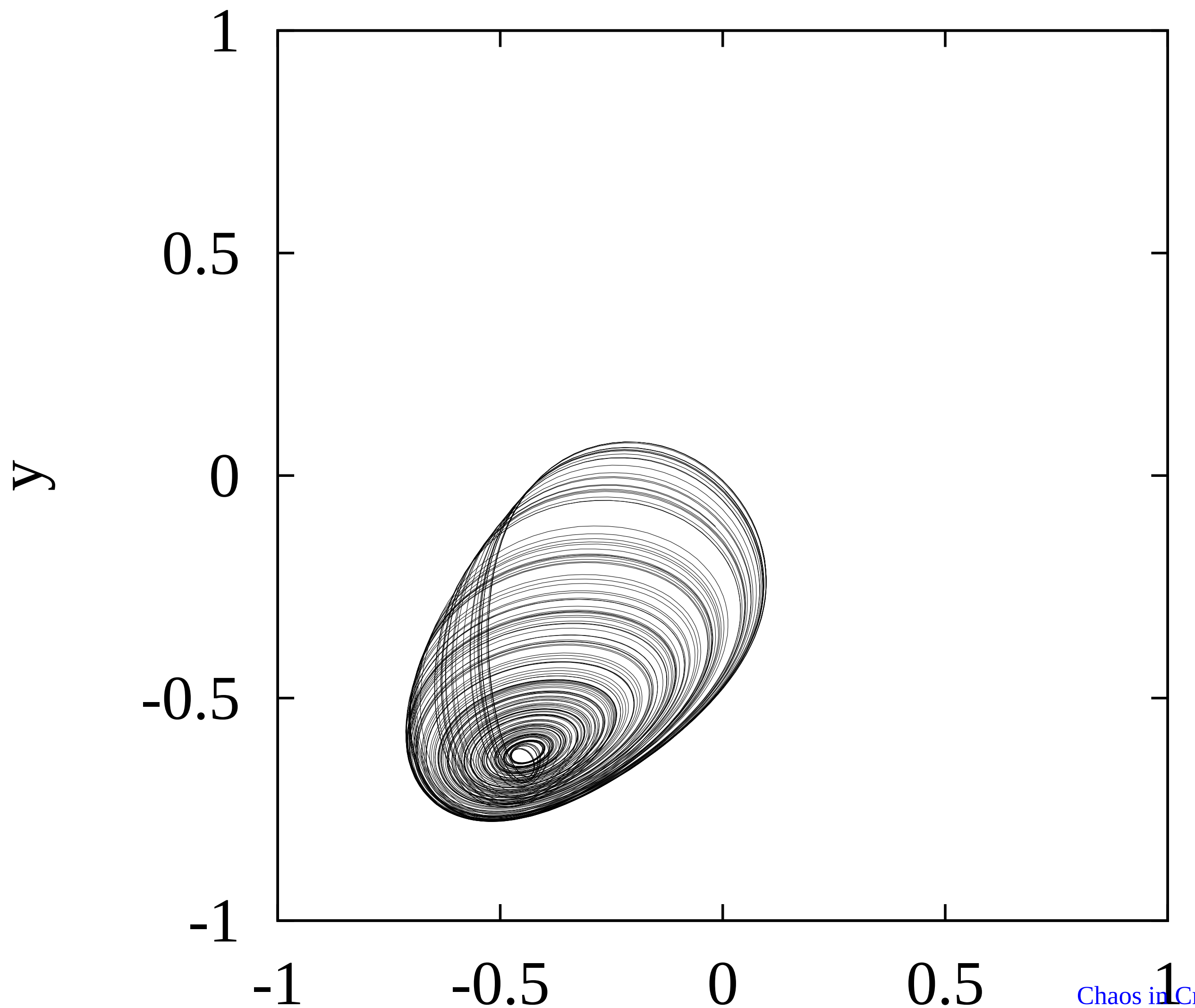

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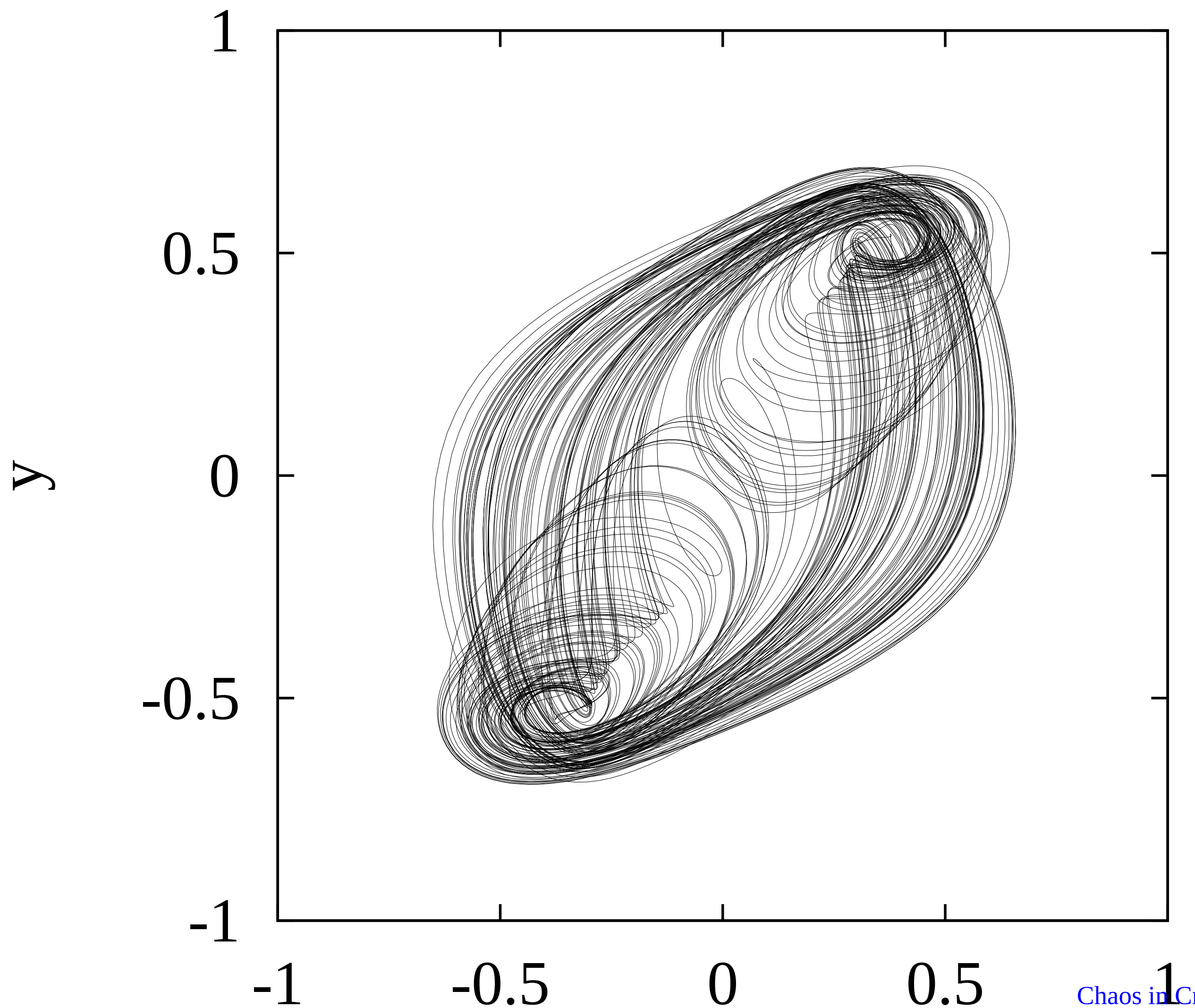
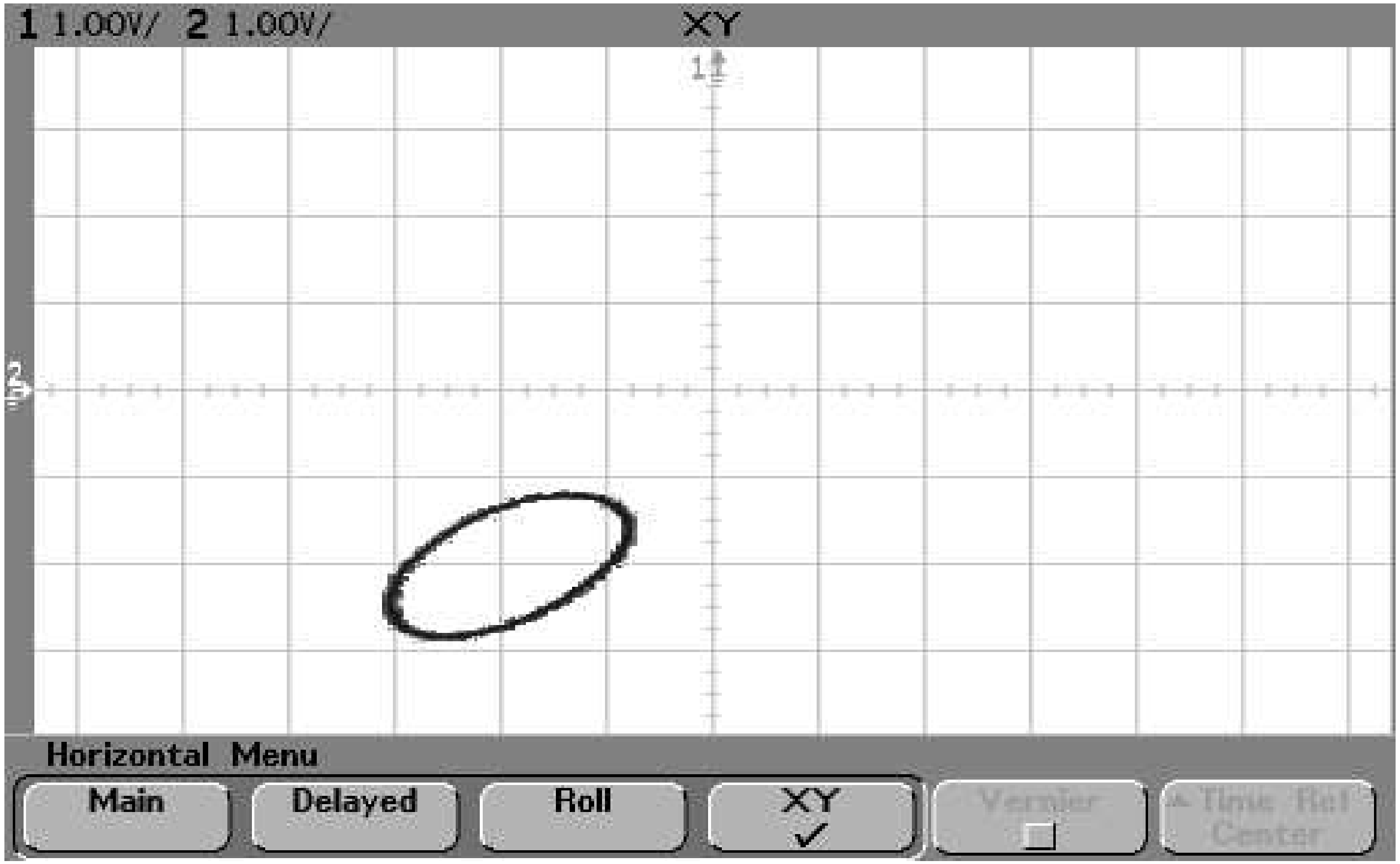
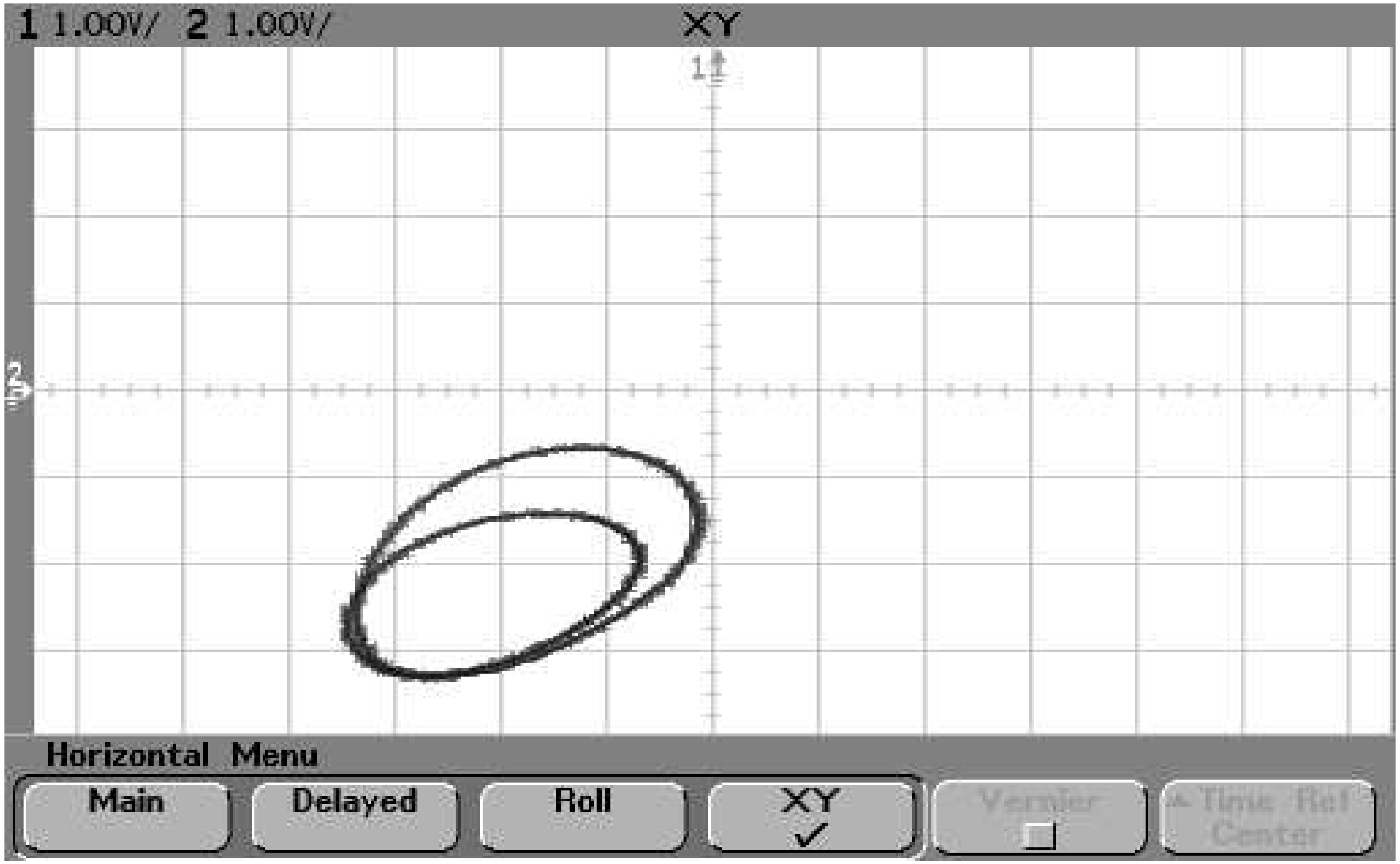


Lab. experiments



$r_2 = 2000$**[Ω],** $G_1 = 1/2000$**[℧],** $G_2 = 1/250$**[℧].** r_2 **is ranged from 216–500 [Ω].**

Lab. experiments



$r_2 = 2000[\Omega]$, $G_1 = 1/2000[\mho]$, $G_2 = 1/250[\mho]$. r_2 **is ranged from 216–500 [Ω].**

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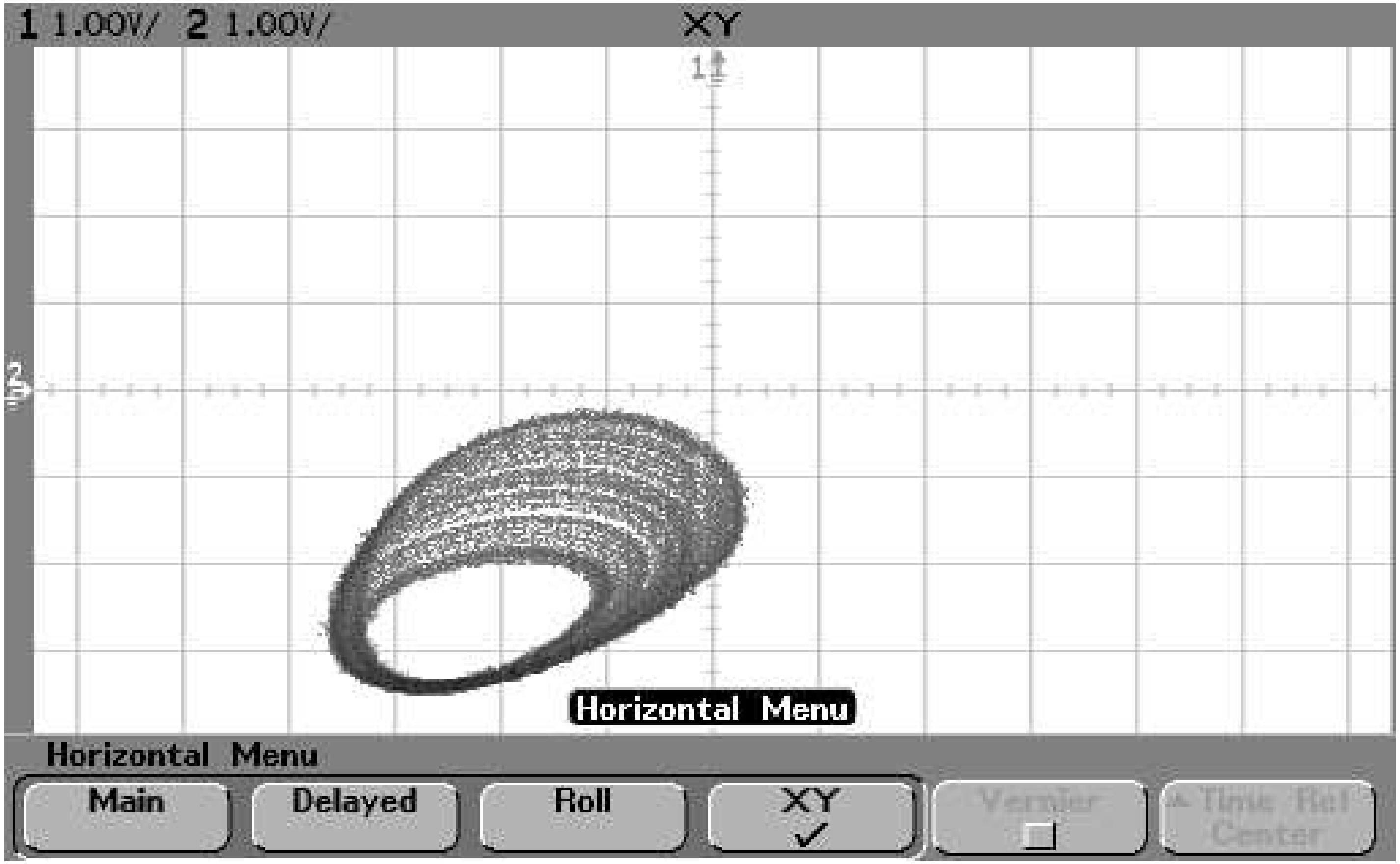

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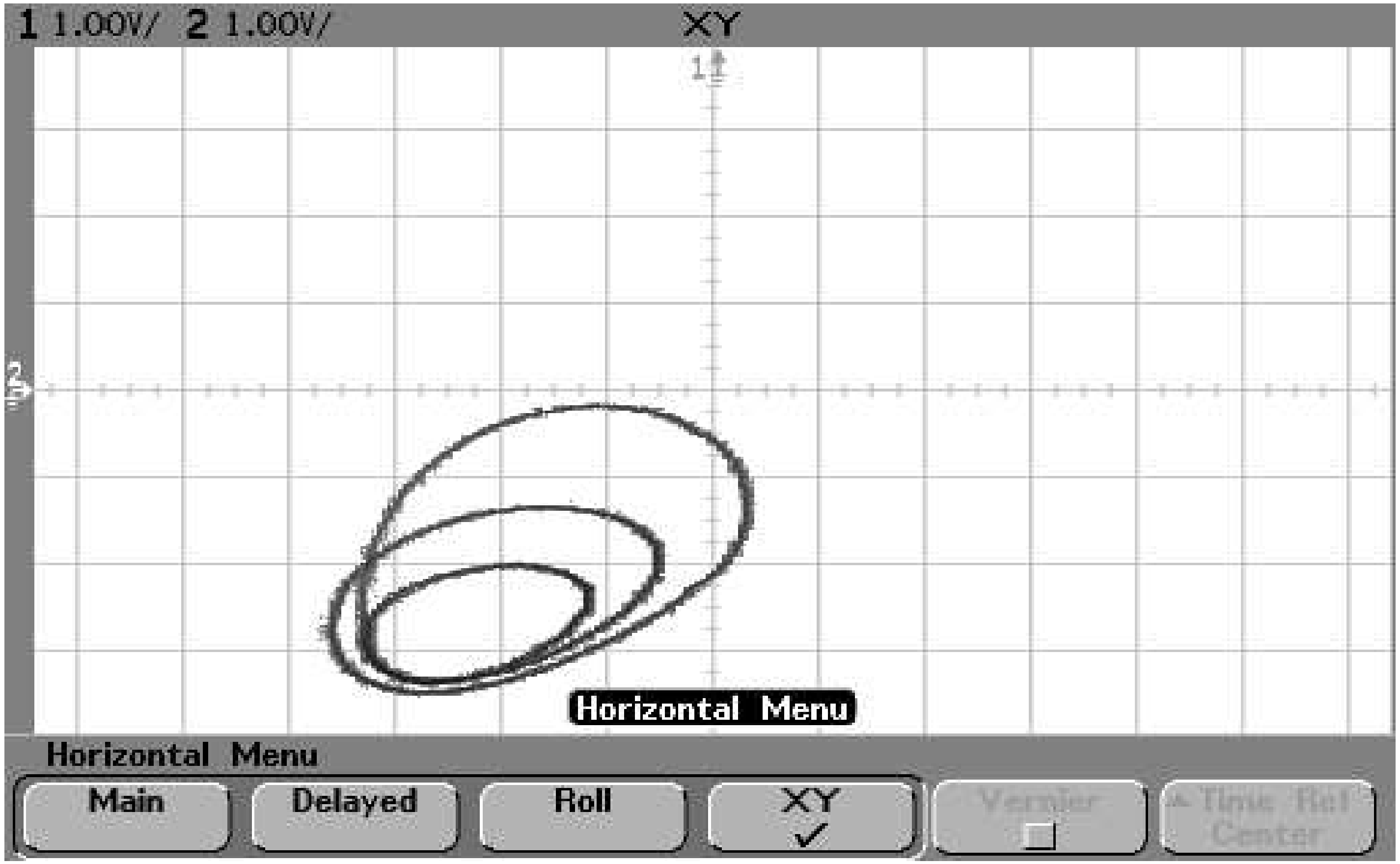

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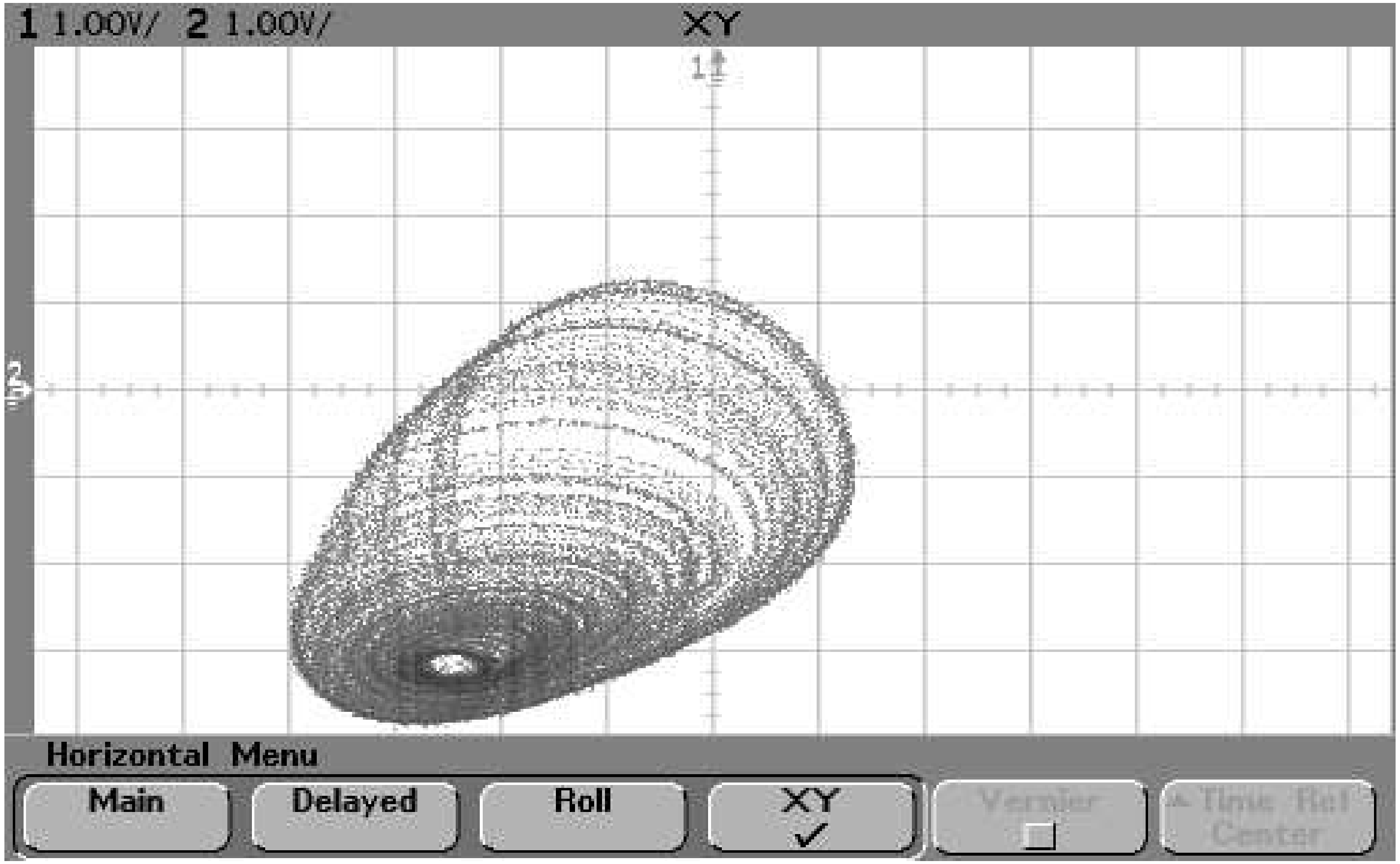

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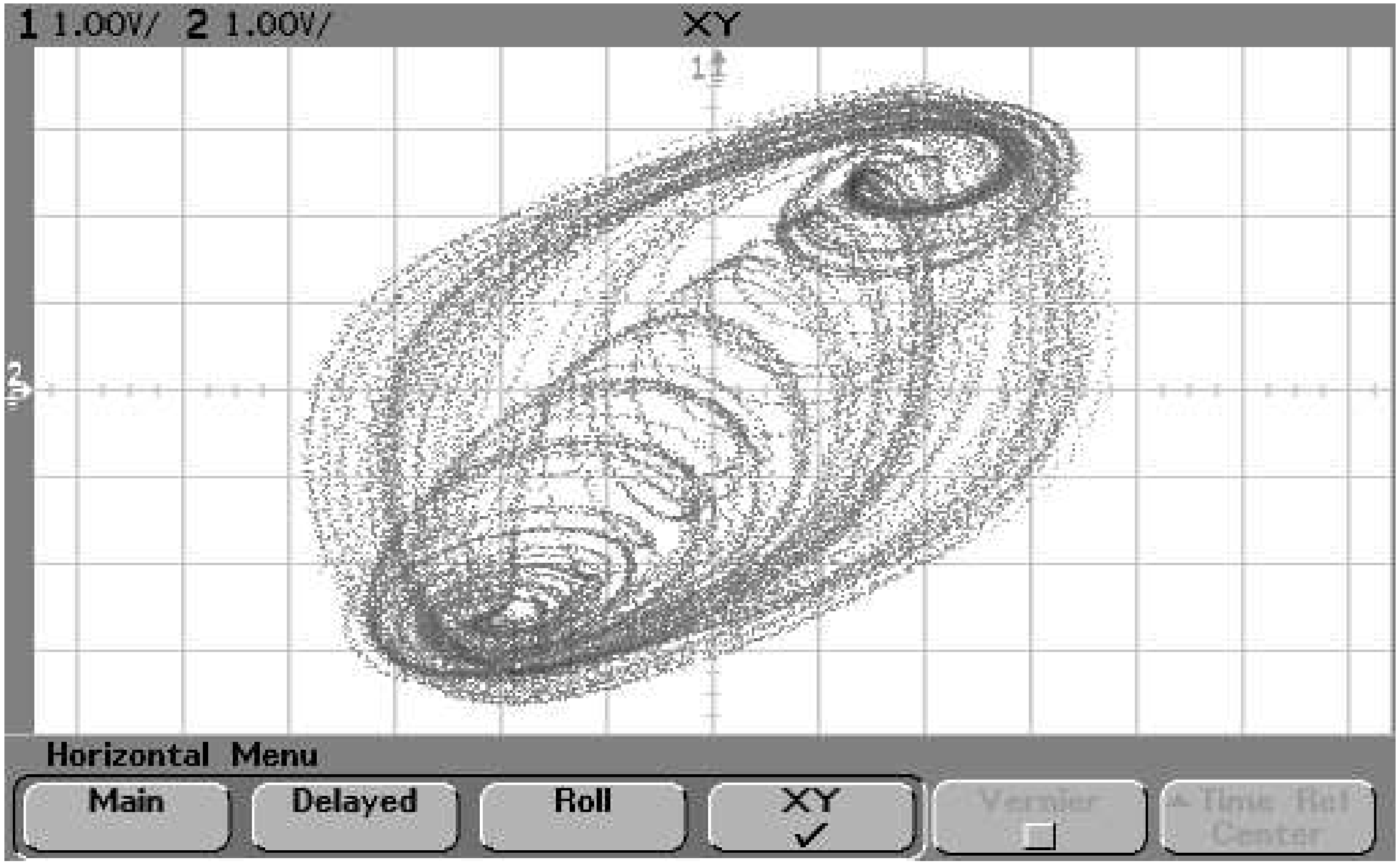
Lab. experiments



$r_2 = 2000$[Ω], $G_1 = 1/2000$[℧], $G_2 = 1/250$[℧]. r_2 is ranged from 216–500 [Ω].

Conclusions

Cross-coupled BVP oscillators

- ✎ **Classification of synchronization mode**
- ✎ **Torus doubling and chaos**
- ✎ **Period doubling cascade**
- ✎ **Circuit realization**