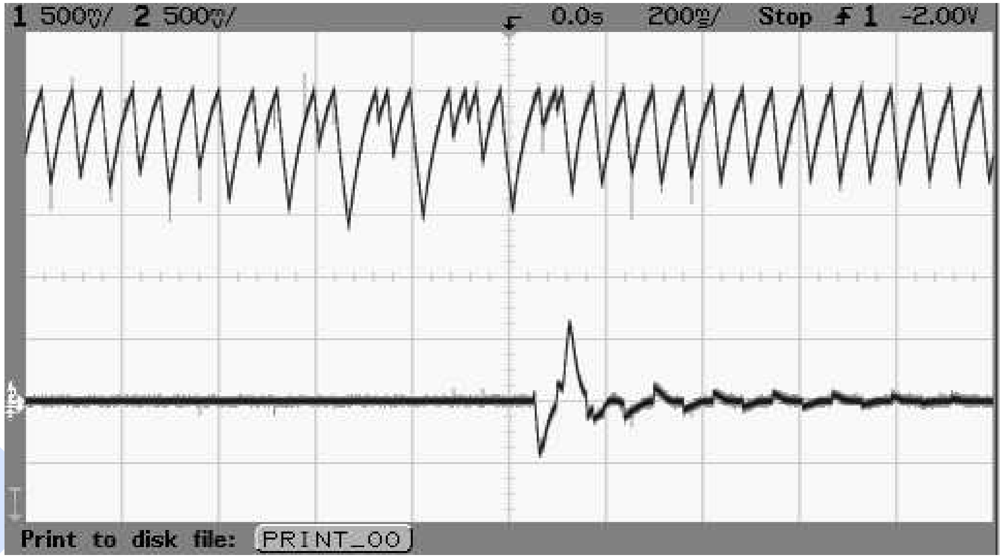
Partial Delayed Feedback Control and its DSP Implementation



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Background

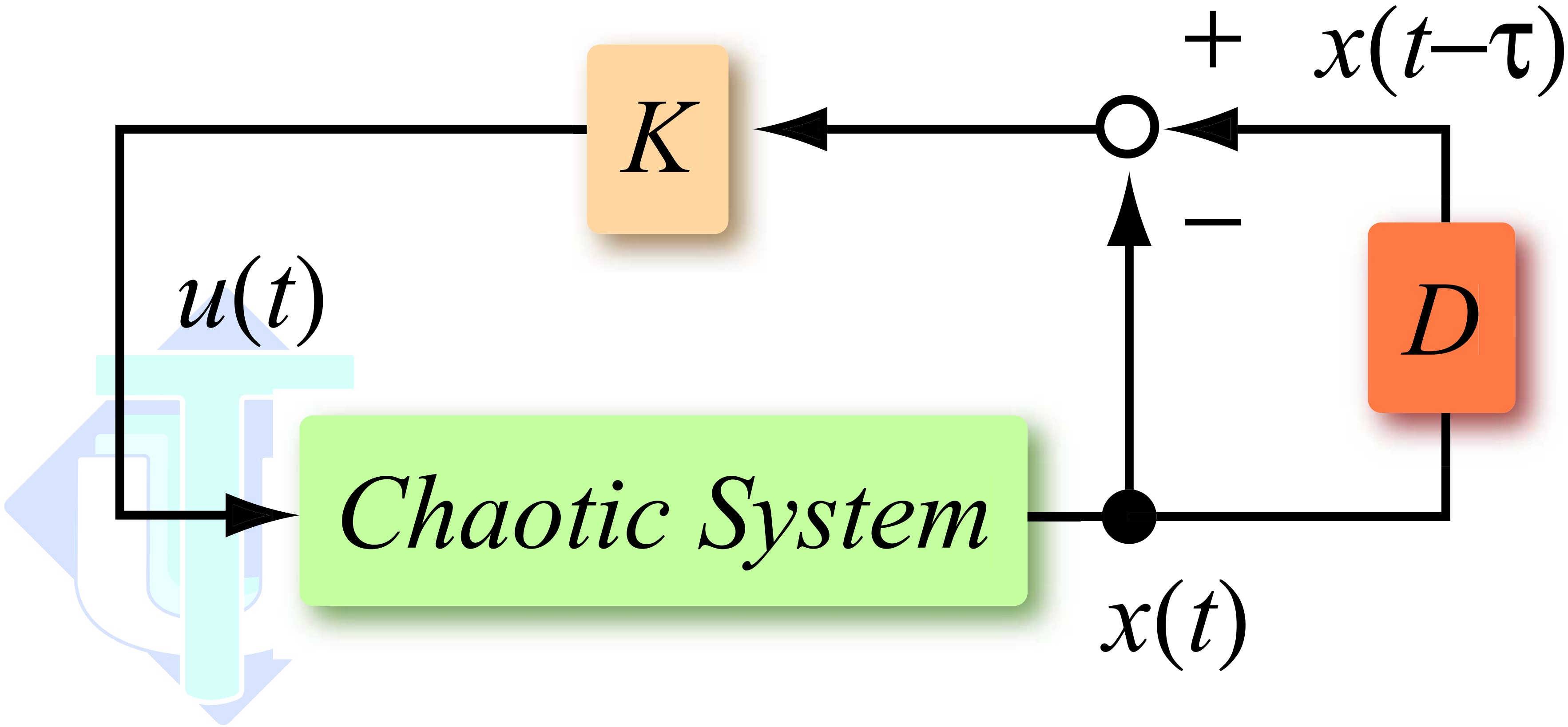
Controlling chaos — Stabilizing an unstable periodic orbit in given nonlinear dynamical system

- **OGY method**
- **pole assignment technique**
- **delayed feedback control (DFC)**
- **external force control (EFC)**

K. Pyragas, "Continuous control of chaos by self-controlling feedback," Phys. Lett, 170A, pp. 421–428, 1992

DFC — Delayed feedback control

Control input: **a difference between the present state and the τ-delayed state**



Background

Delayed Feedback Control(DFC)

Merit:

- **Simple structure**
- **No information about UPOs needed**
- **Can be a UPO detector**

Demerit:

- τ **— unknown for autonomous systems**
- **infinite dimension**
- **difficult to analyze its stability**

Hybrid systems

- **Dynamical systems including interrupts, switches, impulsive inputs**
- **showing chaotic behavior frequently**

Controlling chaos in hybrid systems:

T. Kousaka, S. Tahara, T. Ueta, M. Abe, and H. Kawakami, "Chaos in a simple hybrid system and its control," IEE Electronics Letters, Vol.37, No.1, pp.1 2, Jan.2001

using an OGY-like method.
DFC for chaotic hybrid system is not given !

In this talk...

- Partial Delayed Feedback Control (PDFC)
- PDFC by a digital signal processor (DSP)
- EFC after DFC — stability analysis
- possibly it is reasonable scheme for hybrid systems

Descriptions

A chaotic hybrid system:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(t, \boldsymbol{x})$$

where, $\boldsymbol{x} \in \boldsymbol{R}^n$, $\boldsymbol{f}$: $\boldsymbol{R}^n \to \boldsymbol{R}^n$
Assume that a solution $\boldsymbol{x}(t) = \boldsymbol{\varphi}(t, \boldsymbol{x}_0)$ **would be** *chaos*.
A UPO:

$$\boldsymbol{x}_0 = \boldsymbol{\varphi}(0, \boldsymbol{x}_0) = \boldsymbol{\varphi}(\tau, \boldsymbol{x}_0)$$

where, τ **is the period of the UPO.**

DFC

Conventional DFC

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(t, \boldsymbol{x}) + \boldsymbol{u}(t)$$

$$\forall t > 0$$

$$\boldsymbol{u}(t) = K(\boldsymbol{x}(t - T) - \boldsymbol{x}(t))$$

PDFC

Partial Delayed Feedback Control(PDFC):

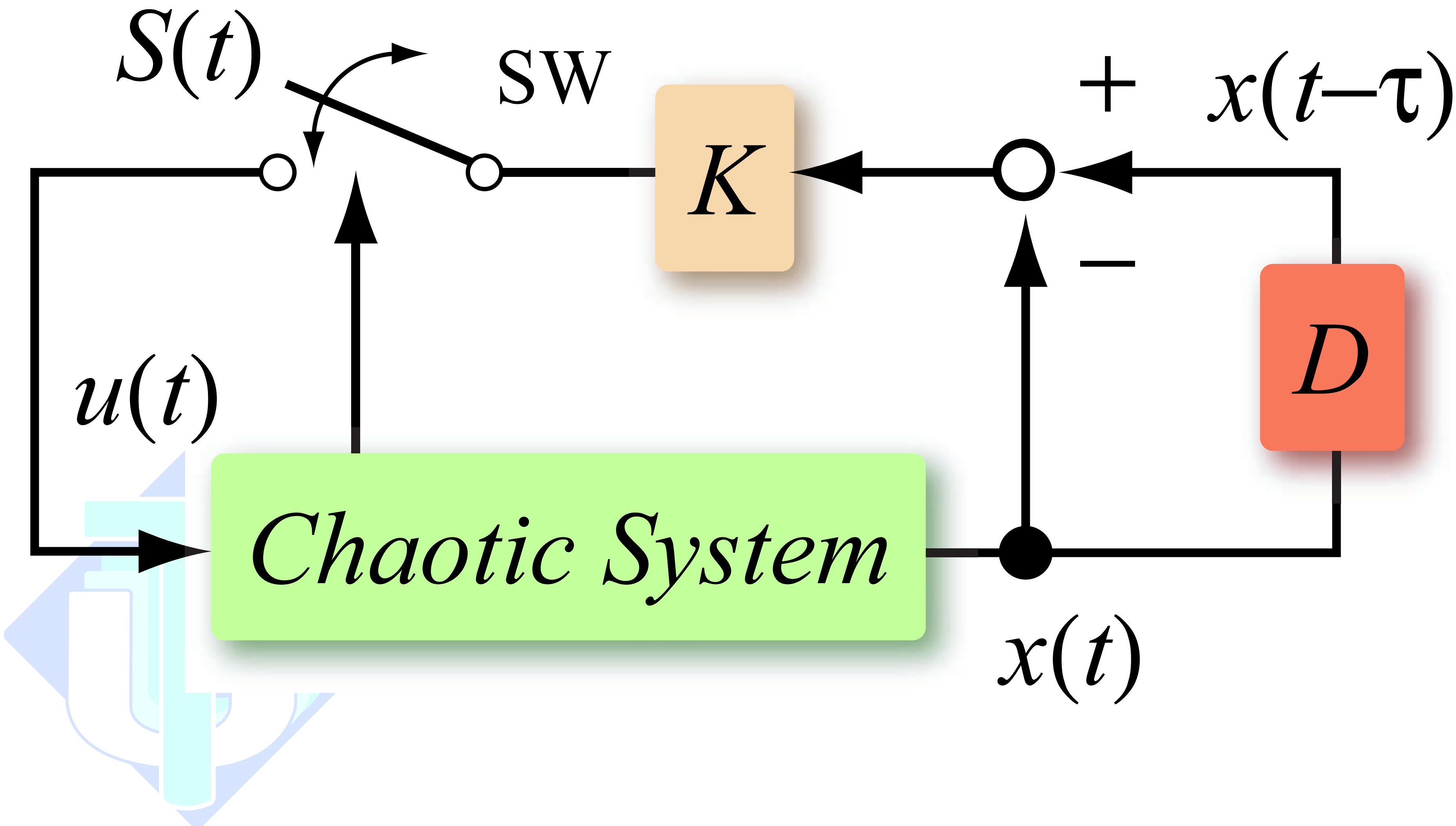
$$\frac{d\boldsymbol{x}}{dt} = f(t, \boldsymbol{x}) + \boldsymbol{u}(t) \qquad \text{if} \qquad C(\boldsymbol{x}) \in M$$

$$\frac{d\boldsymbol{x}}{dt} = f(t, \boldsymbol{x}) \qquad \text{otherwise}$$

$$\boldsymbol{u}(t) = K(\boldsymbol{x}(t-T) - \boldsymbol{x}(t))$$

$C(x)$: **condition depending on the state.**

PDFC



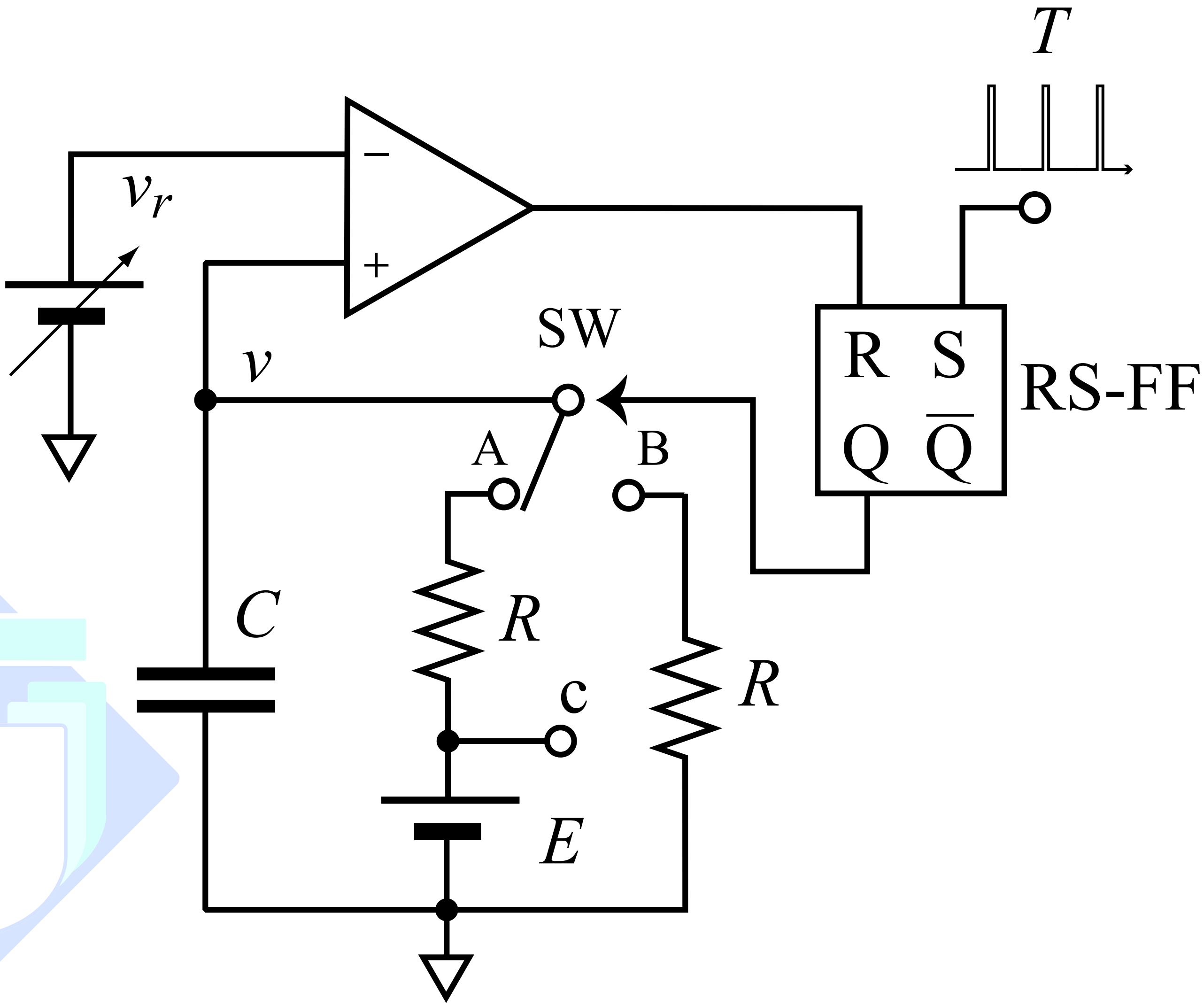
PDFC

- A control input computed from data stored in a memory is applied to the system intermittently.
- The control scheme may be “cutting corners,” “natural,” or “手抜き” — “follow the system as is.”
- possibly effective to: circuits having a restricted timing for control or a refractory period, robots including passive joints, etc.

Related works

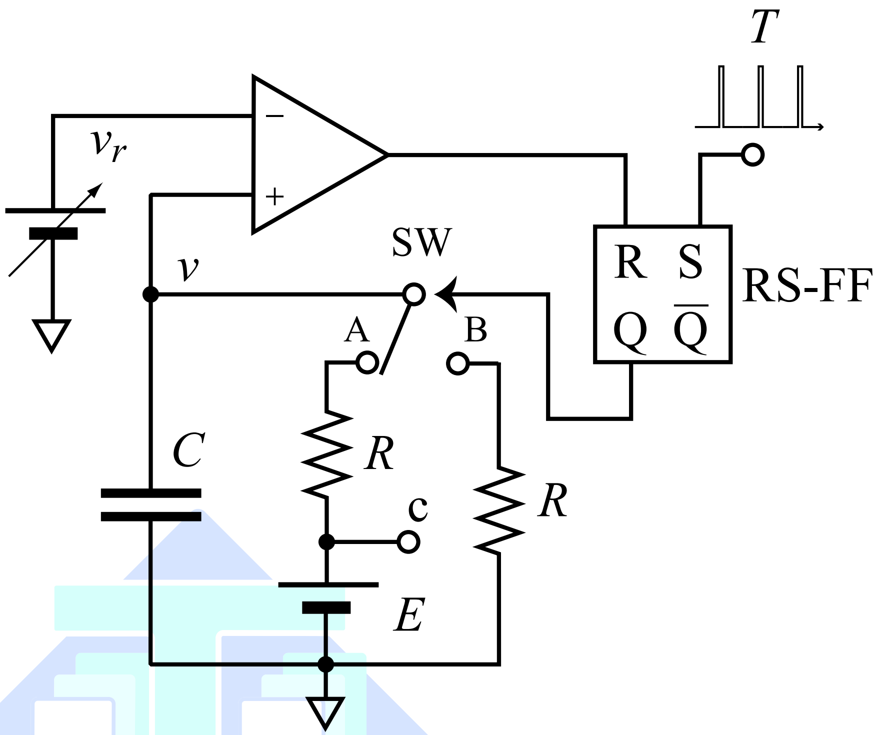
- **P. Hovel and J. E. S. Socolar, "Stability domains for time-delay feedback control with latency," Phys. Rev. E 68, 036206 (2003) The DFC input latency is discussed.**
- **K. Myneni, T. A. Barr, N. J. Corron, and S. D. Pethel, "New method for the control of fast chaotic oscillations," Phys. Rev. Lett. 83, 2175-2178 (1999) Partial signal input is applied as an open-loop control.**

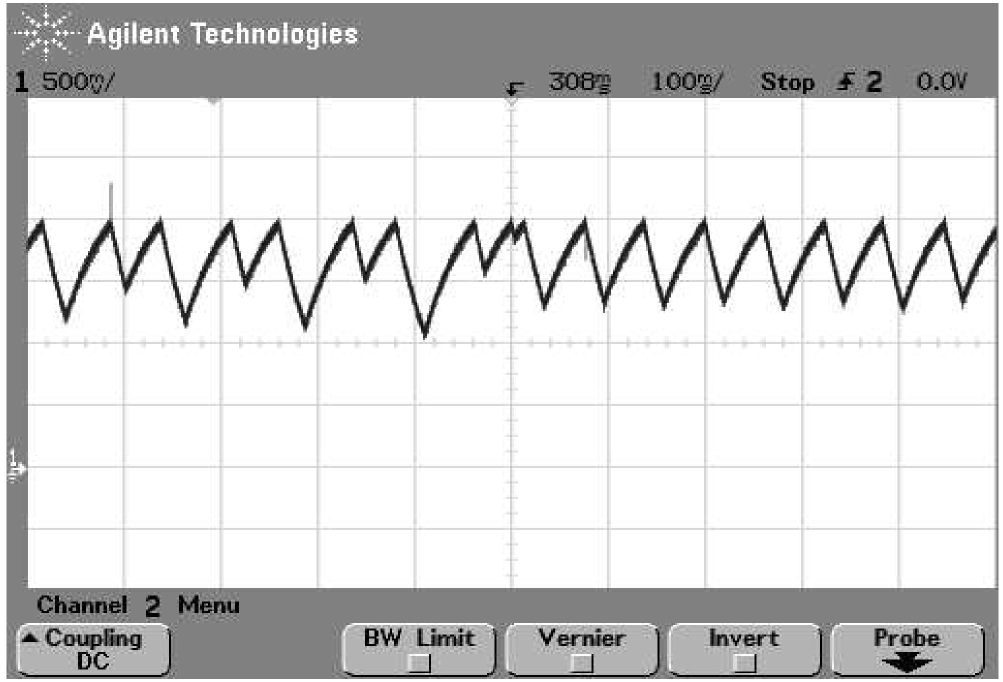
PDFC for a simple chaotic system



T. Kousaka, et al., IEEJ Vol.122-C, No. 9, 2002.

Behavior 1/4

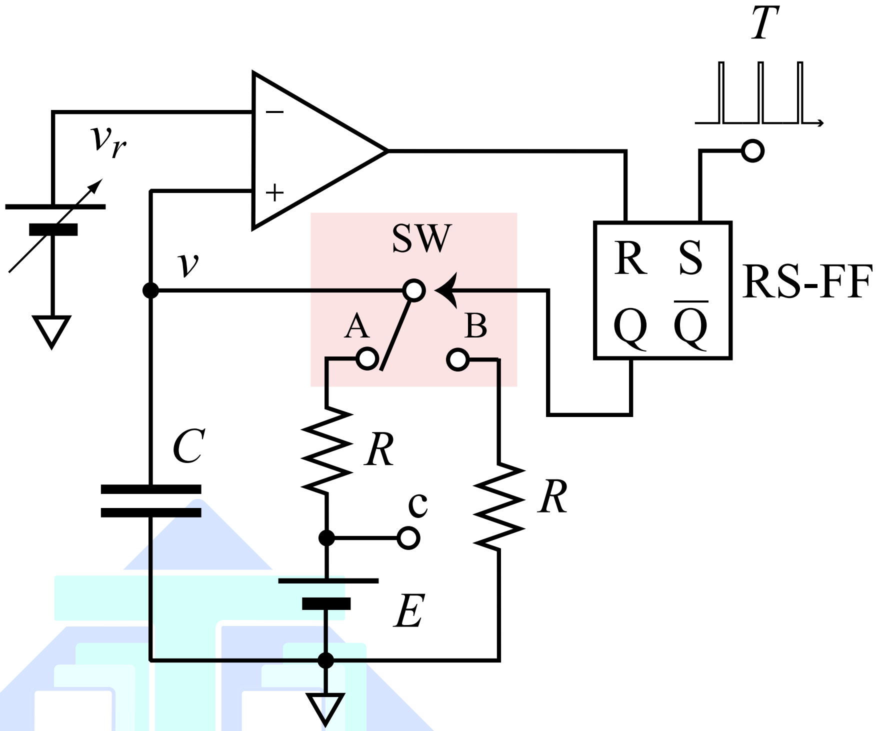


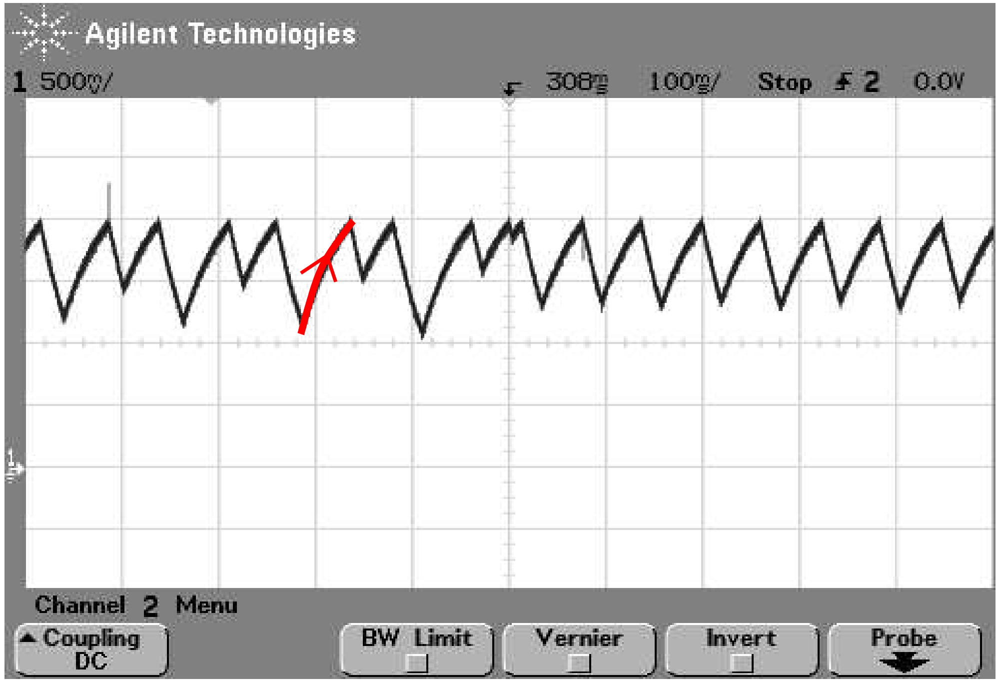


A wave form of the capacitor voltage v

Behavior 2/4

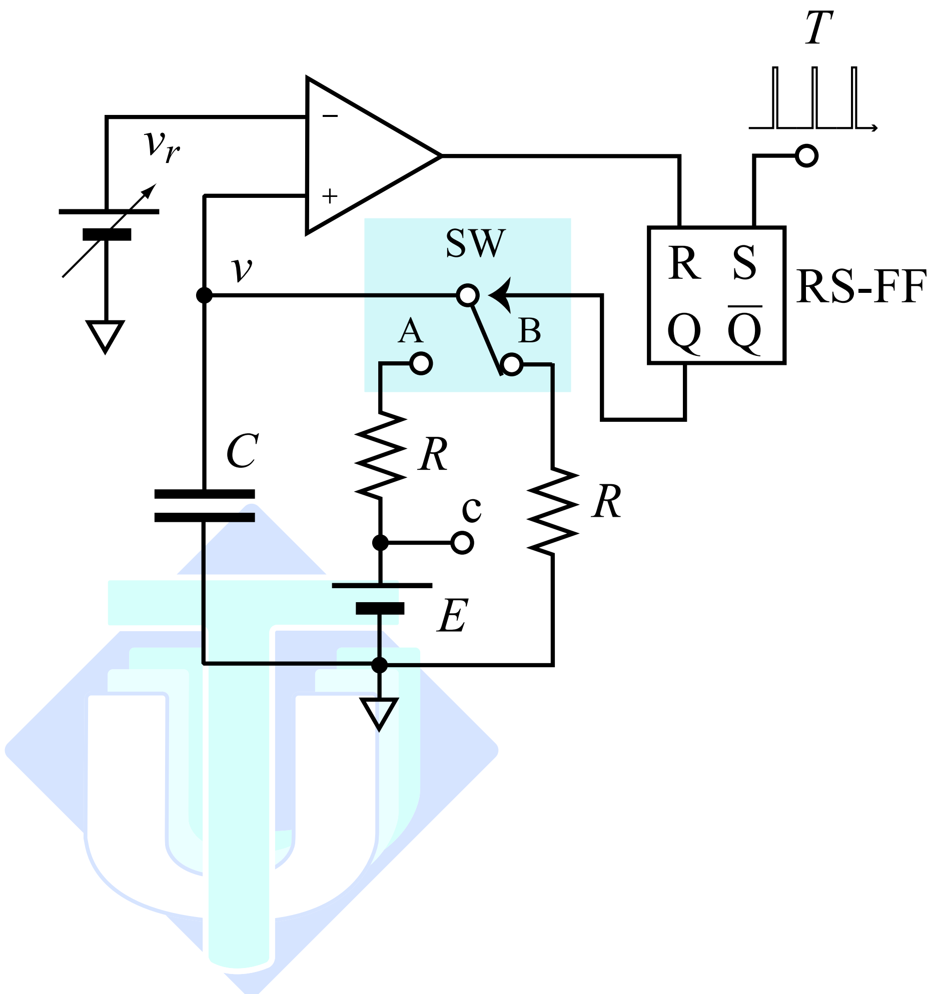
switch position: A

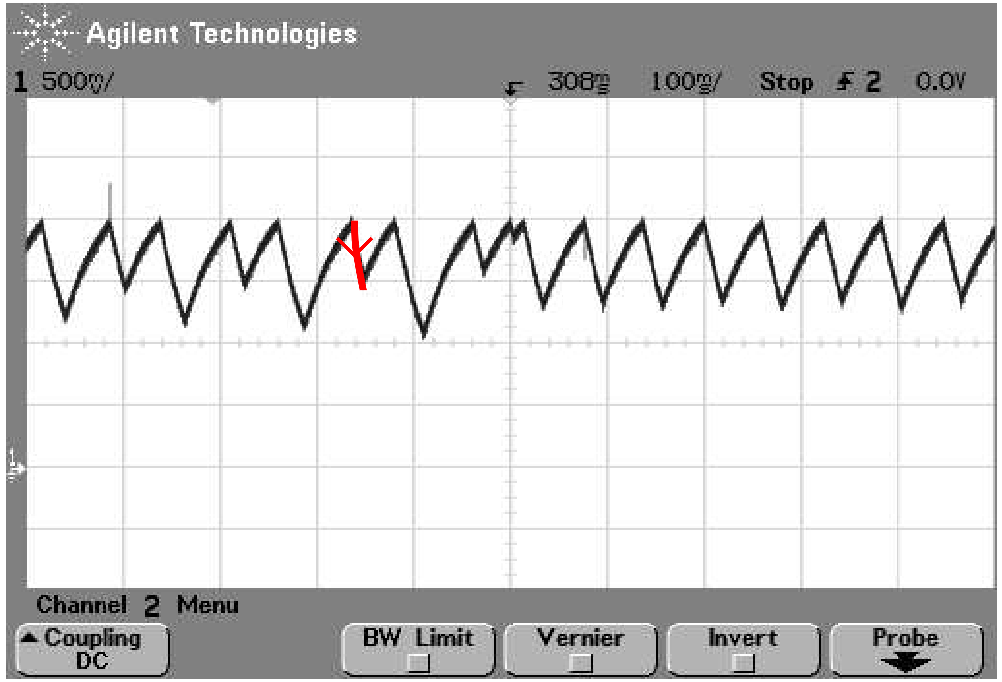




Behavior 3/4

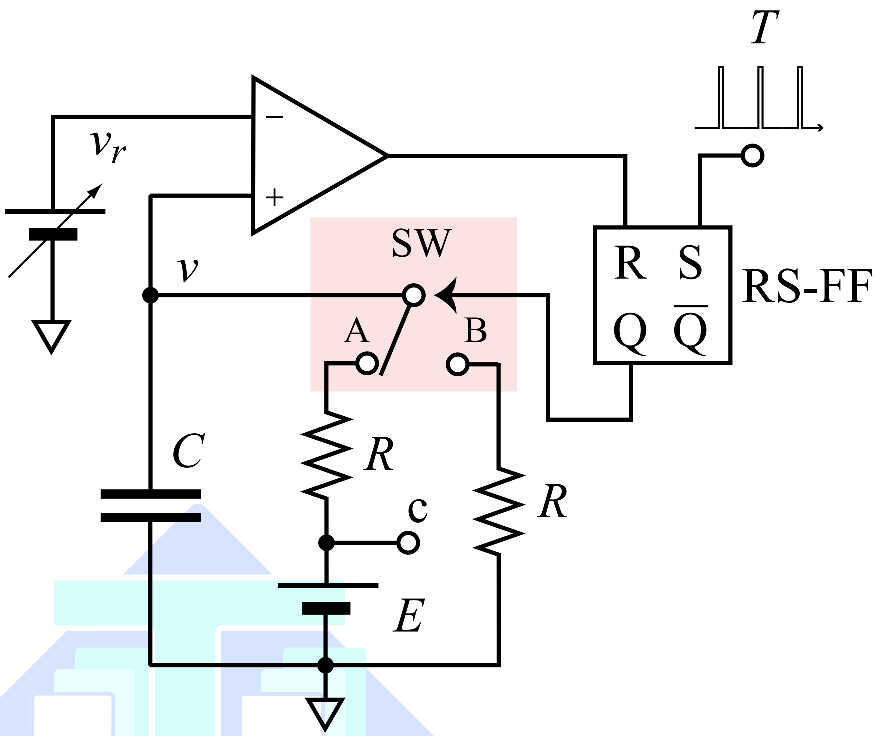
switch position: B

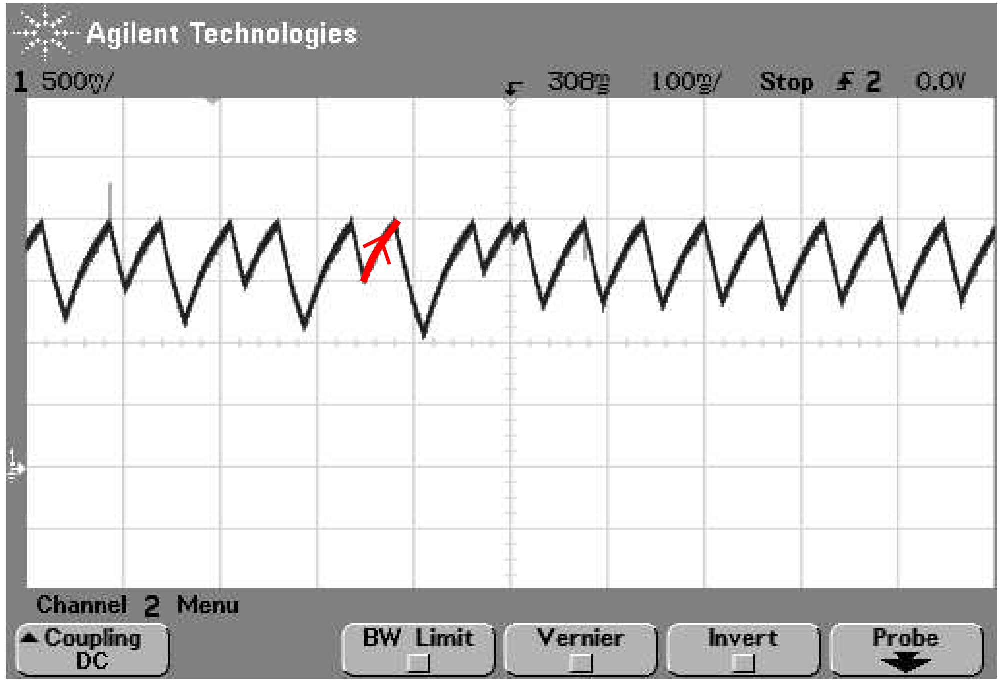




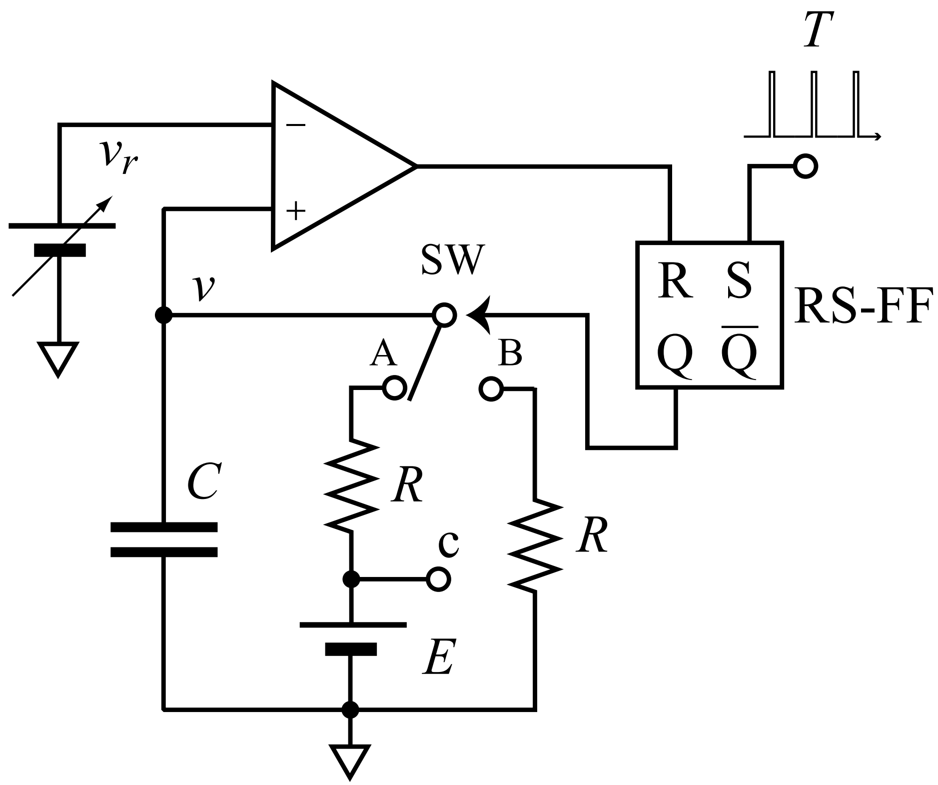
Behavior 4/4

switch position: A





Chaos: real parameters

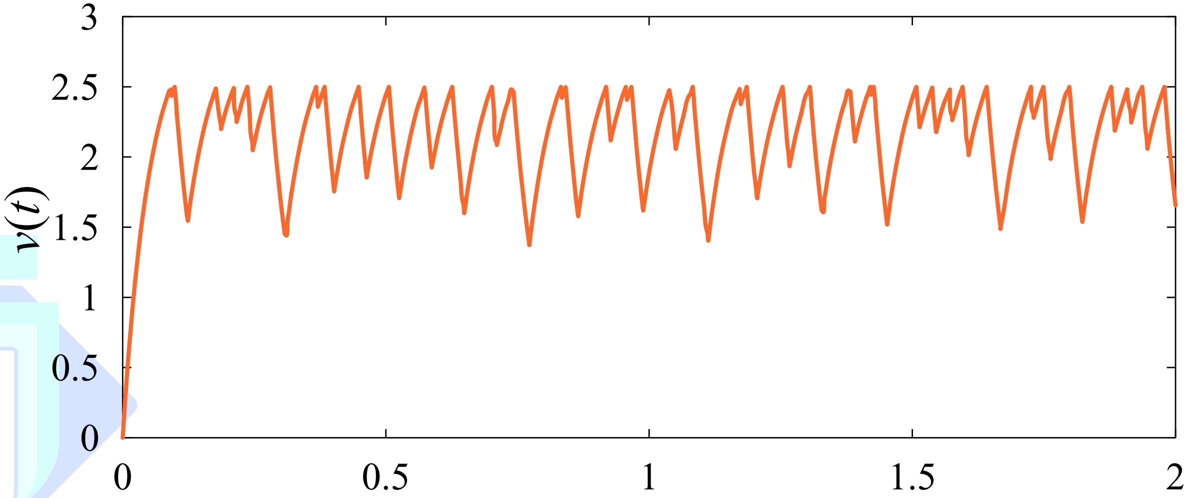


$v_r = 2.5$[V], $R = 51$[kΩ], $C = 1.0$[μF], $E = 3.0$[V],
pulse period: $T = 30$**[msec].**

Circuit equation and simulation

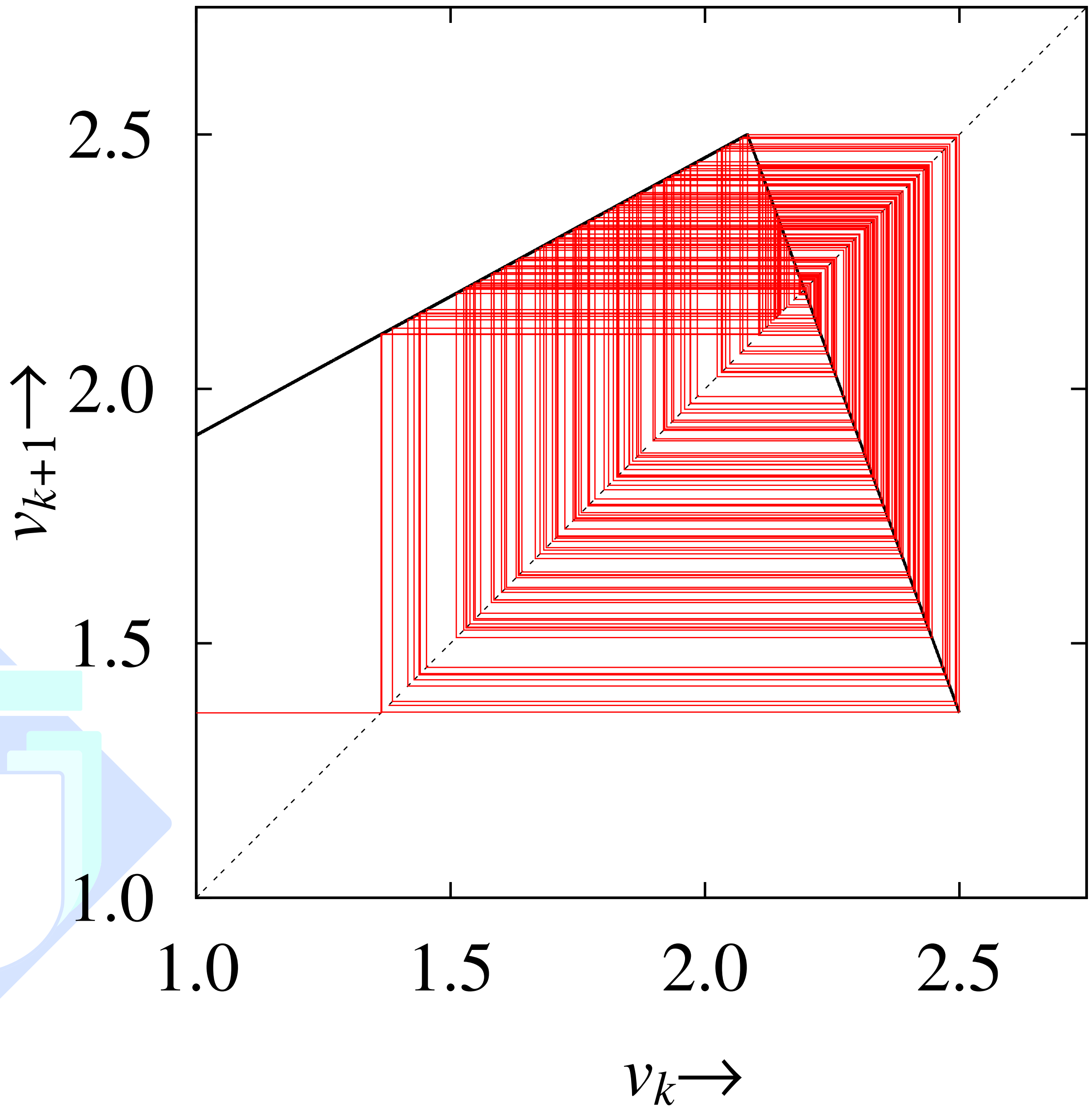
$$\frac{dv}{dt} = -(v - E) \quad \textbf{if} \quad \textbf{position A}$$

$$\frac{dv}{dt} = -v \quad \textbf{otherwise} \quad \textbf{position B}$$

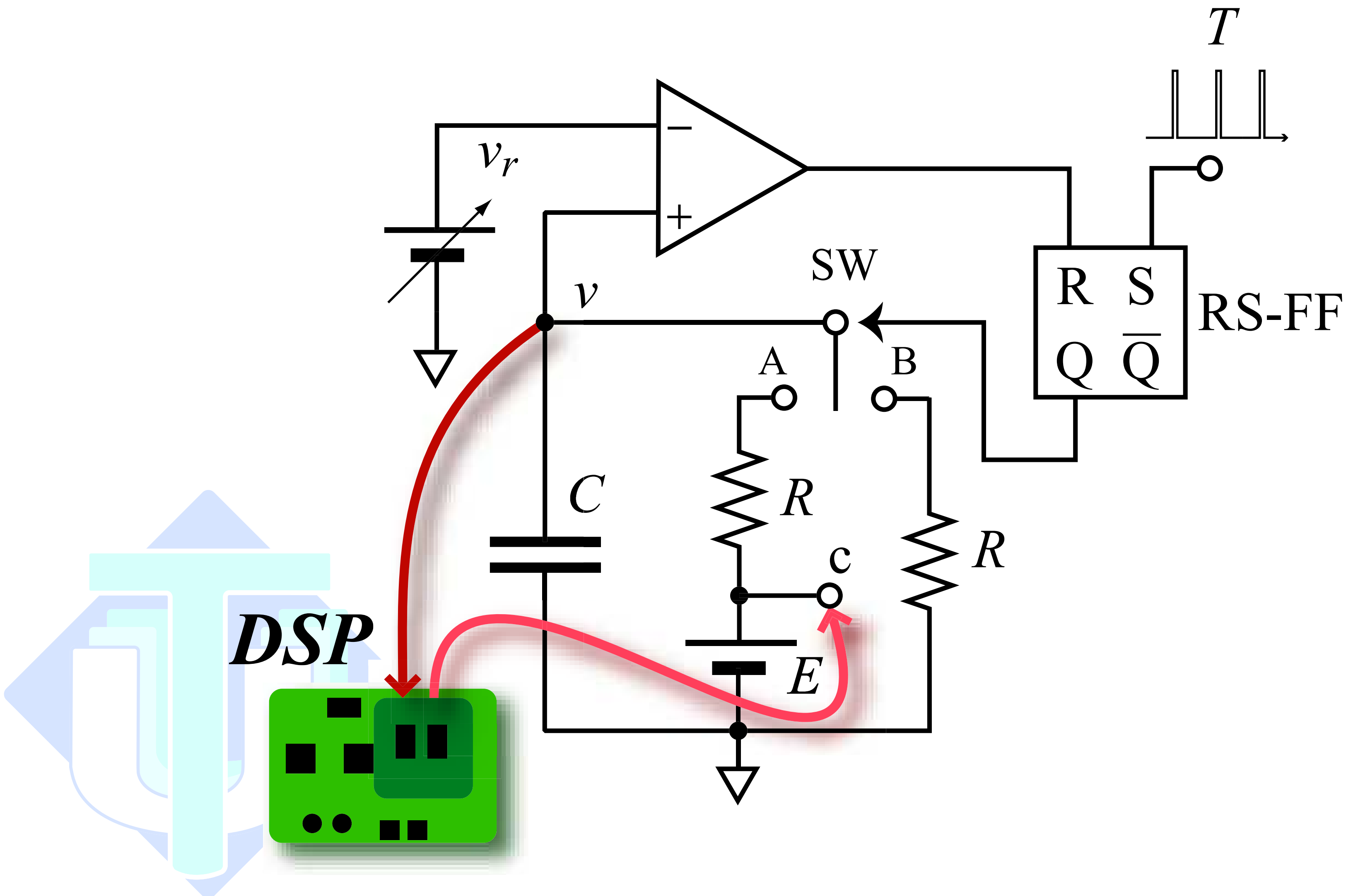


horizontal: t[sec], vertical: v[V]

Return map



Apply PDFC to the circuit



Apply PDFC to the circuit

- The control input is added into the terminal c.
- DSP stores sampled voltage sequence in the ring buffer.
- Delay duration — $\tau = nT, n = 1, 2, \ldots$: pulse period
- Feedback gain K can be controlled by both DSP and analog amplifier.

Controlled system

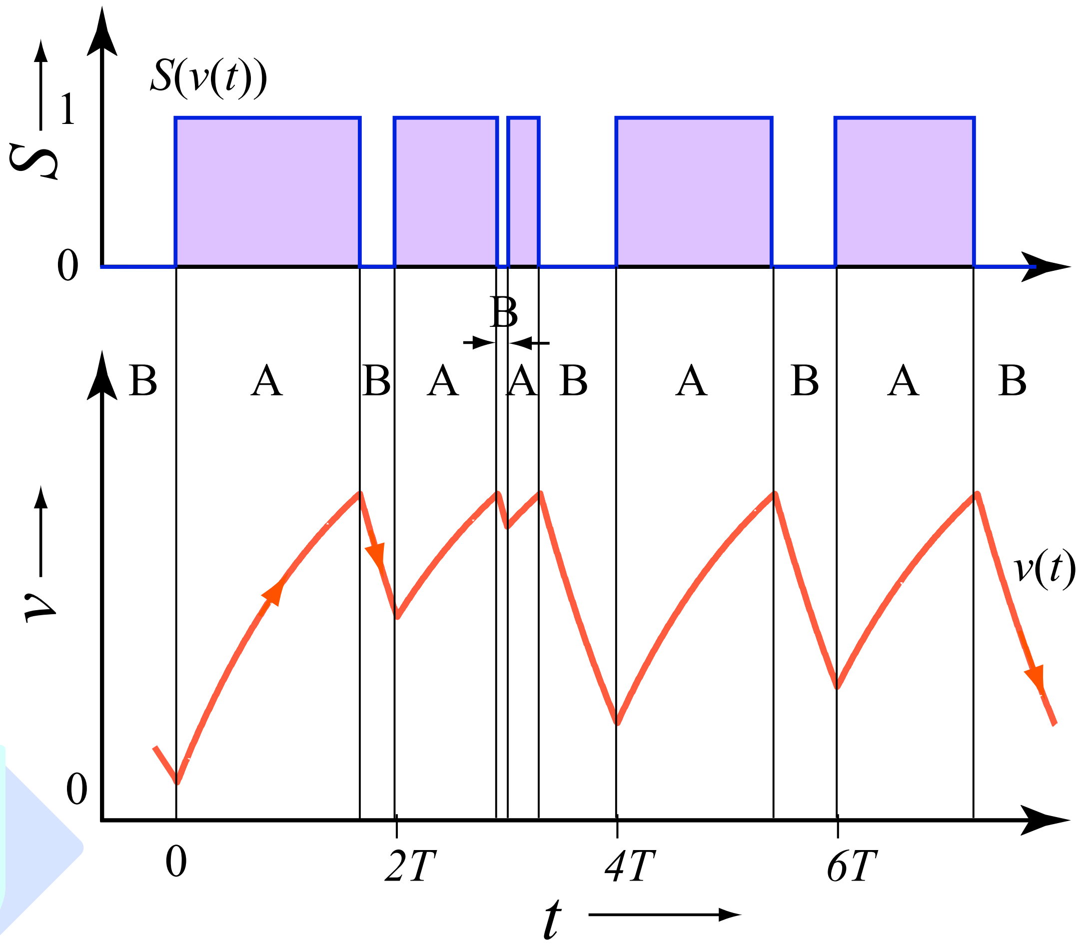
$$\frac{dv}{dt} = -(v - E - u(t)) \quad \text{if} \quad \textbf{position A}$$

$$\frac{dv}{dt} = -v \quad \text{otherwise} \quad \textbf{position B}$$

$$u(t) = S(v(t)) \cdot K(v(t-T) - v(t))$$

$$S(v(t)) = \begin{cases} 1 & \text{if} \quad \textbf{position A} \\ 0 & \text{otherwise} \quad \textbf{position B} \end{cases}$$

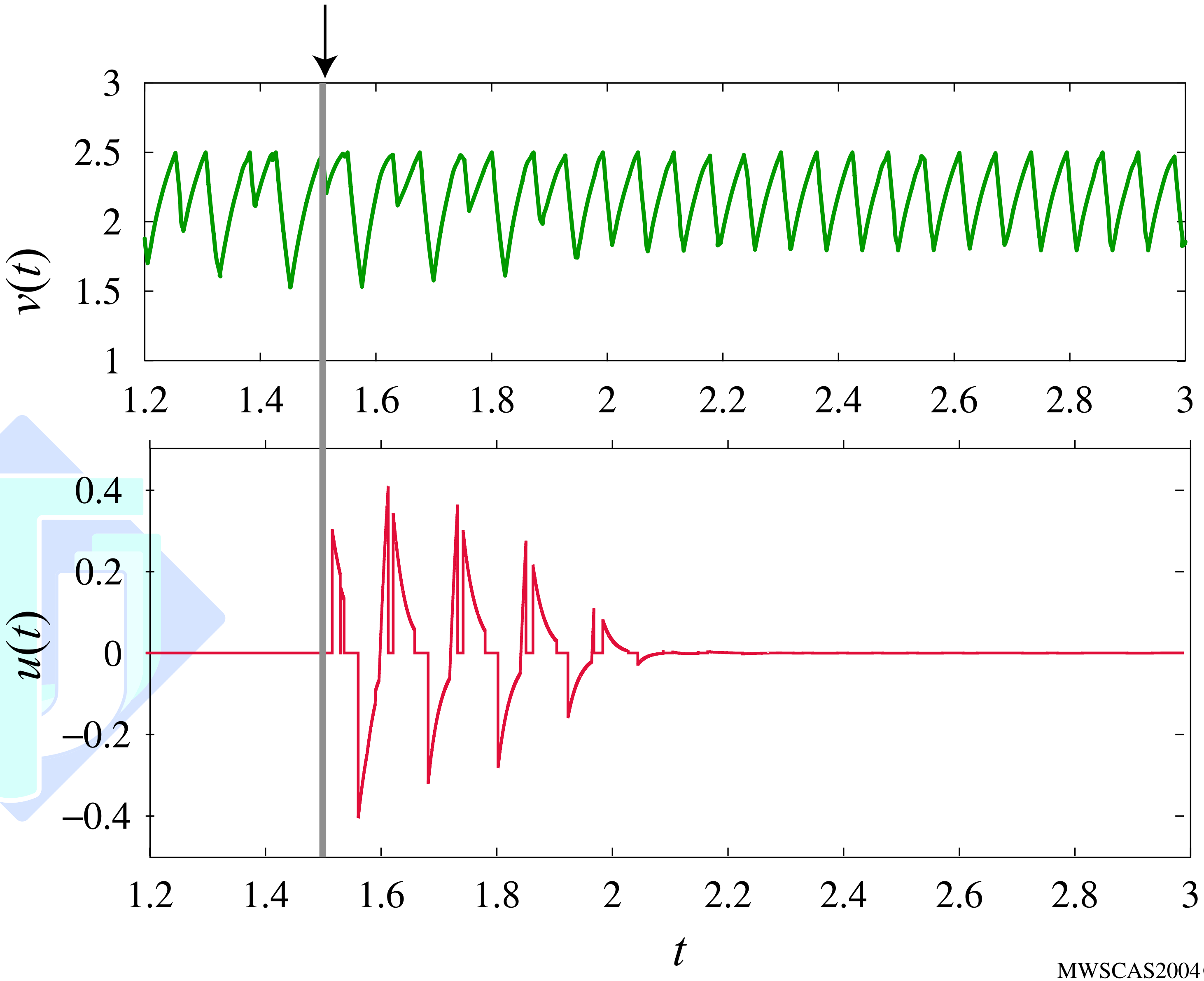
Wave form of $S(v(t))$



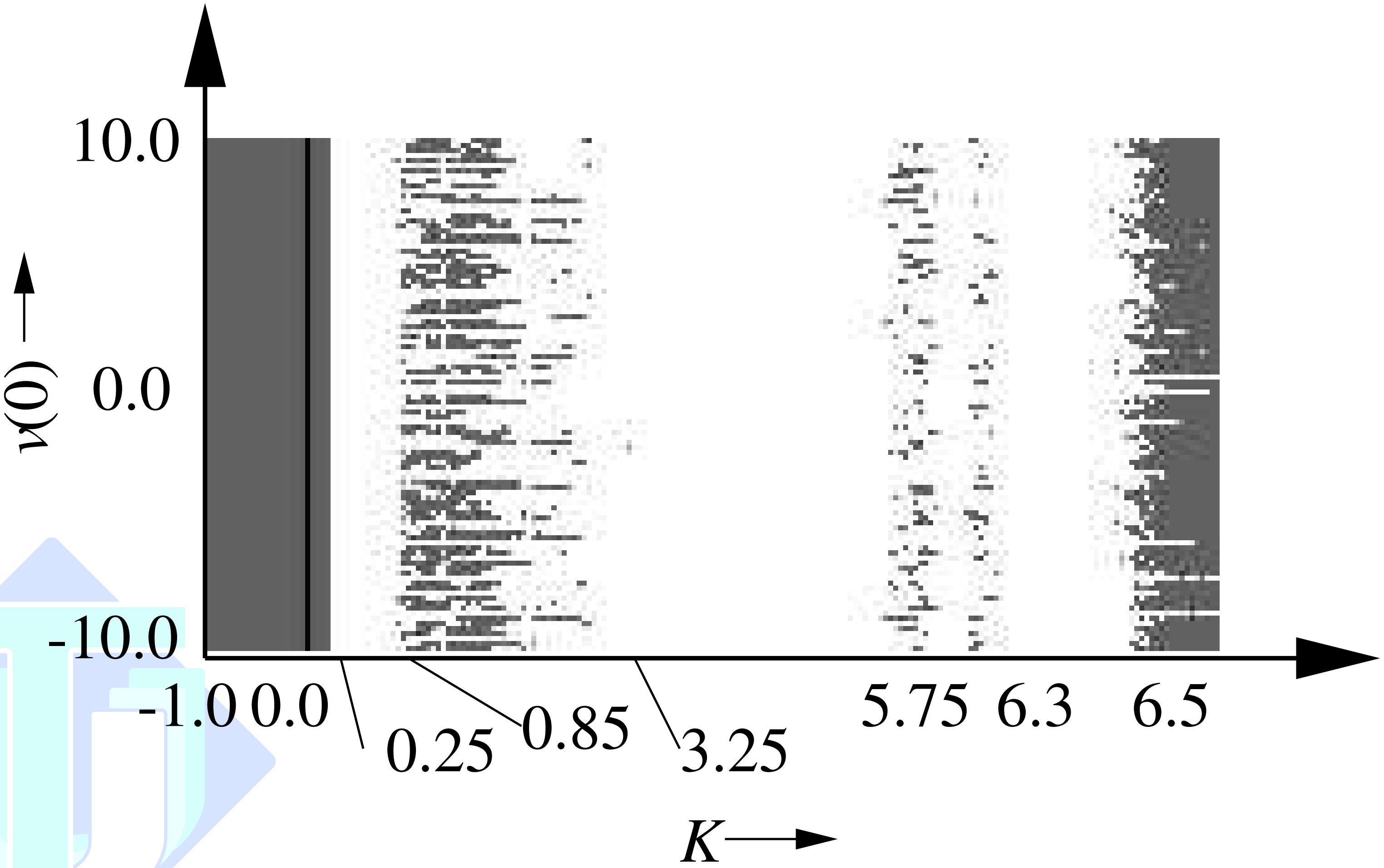
$u(t) = S(v(t)) \cdot K(v(t-T) - v(t))$: **not full energy of $K(v(t-T) - v(t))$ is consumed.**

$K = 0.6, v(0) = 0.0, \tau = 2T$.

The control is enabled at $t = 1.5$[sec].



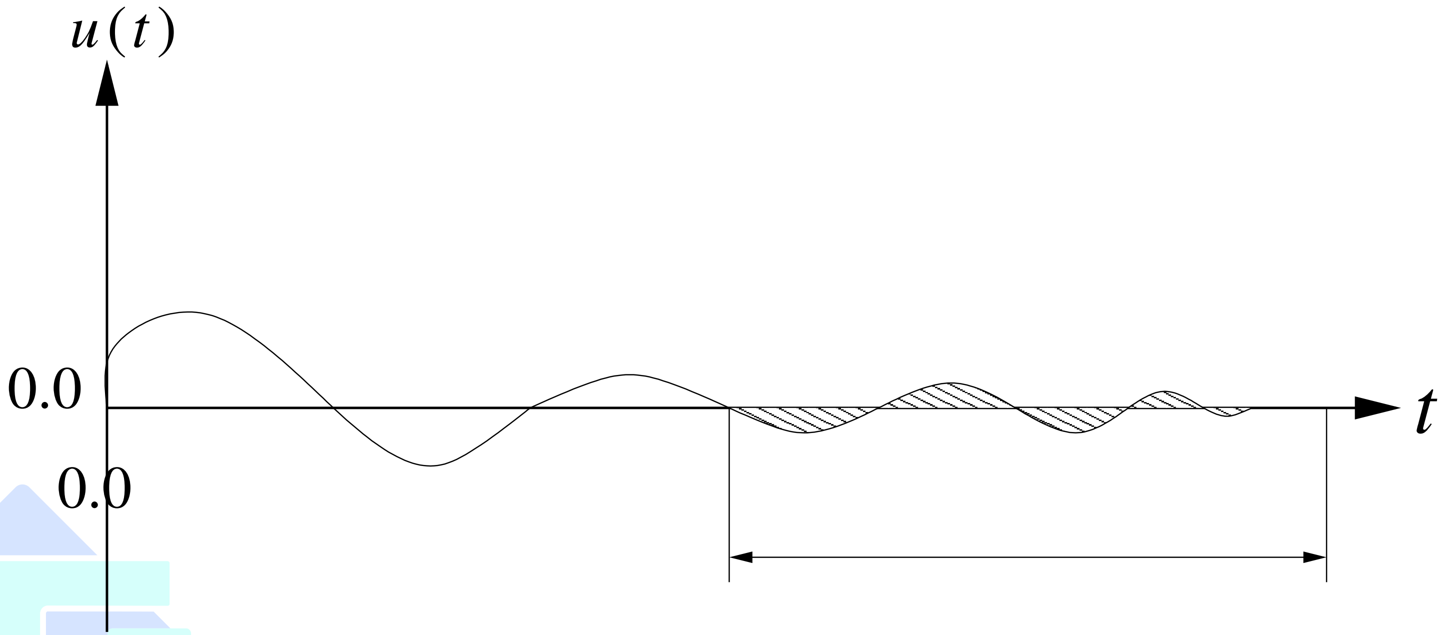
Basin of attraction



The control is enabled from $t > 1.53$ [sec]
White: $u(t)$ converges, Black: $u(t)$ otherwise.

Basin of attraction

Detection of convergence for $u(t)$:



$$\sum_{t=20.4[\mathrm{sec}]}^{25.5[\mathrm{sec}]} |u(t)| < \epsilon,$$

$\epsilon \approx 0.1.$

DSP implementation of PDFC

Texas Instruments, TMS320 C6711DSP + TMDX326040A
16-bit, 44 kHz, stereo I/O.

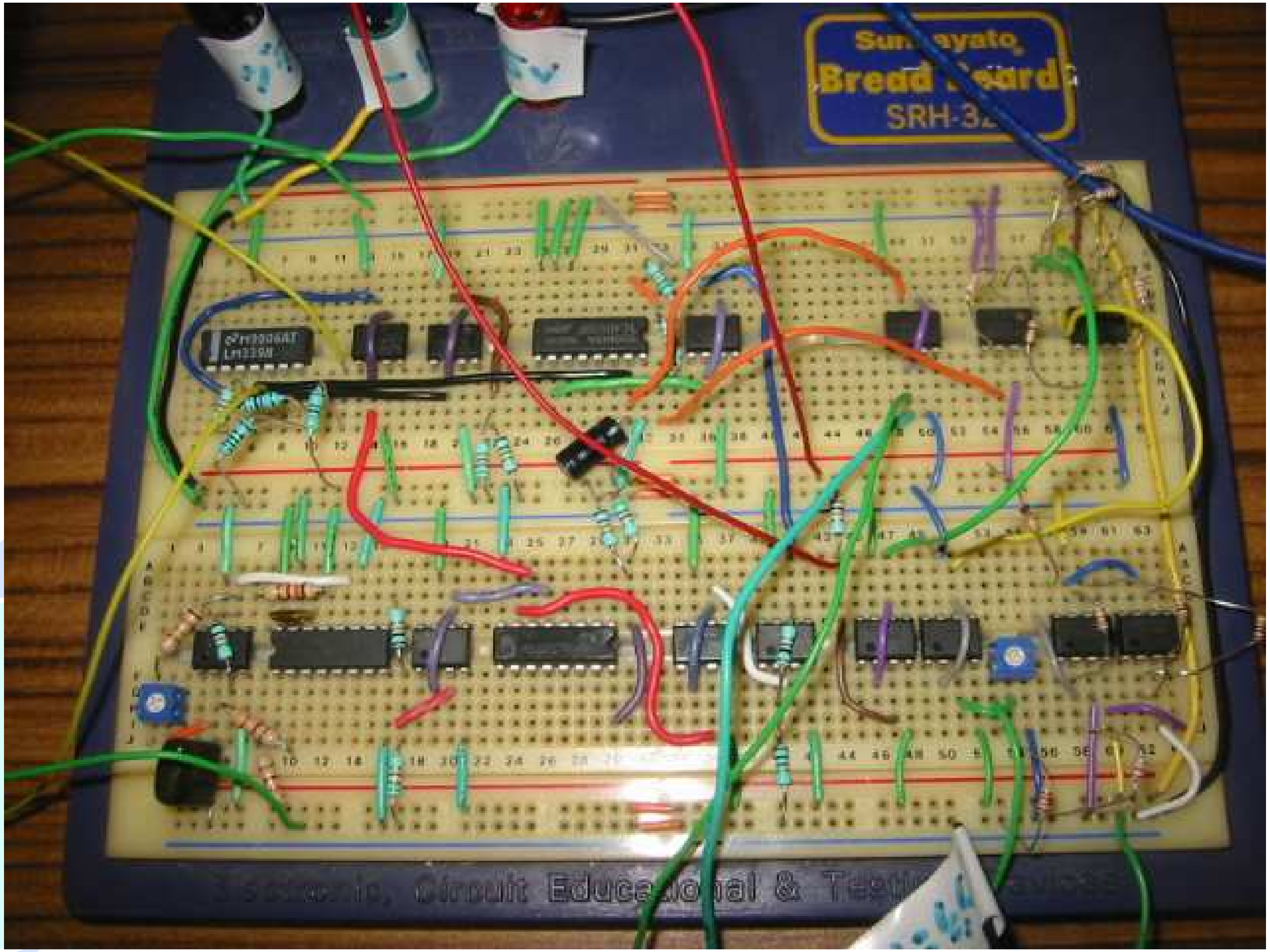
DSP implementation of PDFC

Algorithm on DSP:

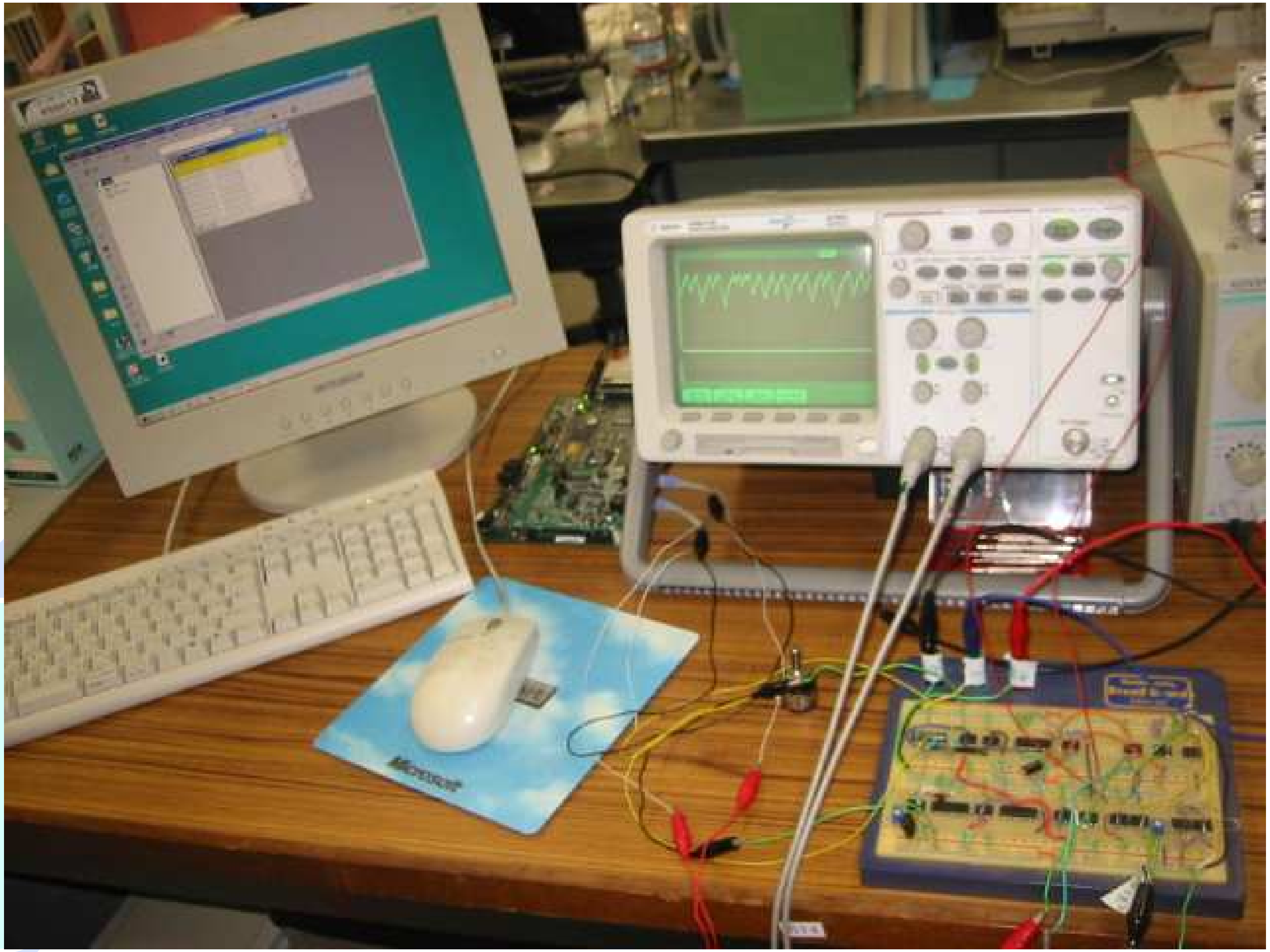
1. **Allocation of a ring buffer whose size is coincident to the sampling rate $\times\,\tau$.**
2. **AD conversion $v(t) \rightarrow x(t)$ and storing as 32bit unsigned integer variable.**
3. **computing $u = x(t-\tau) - x(t)$ and output u.**

Software: C language, CodeComposer v.2.0

DSP implementation of PDFC

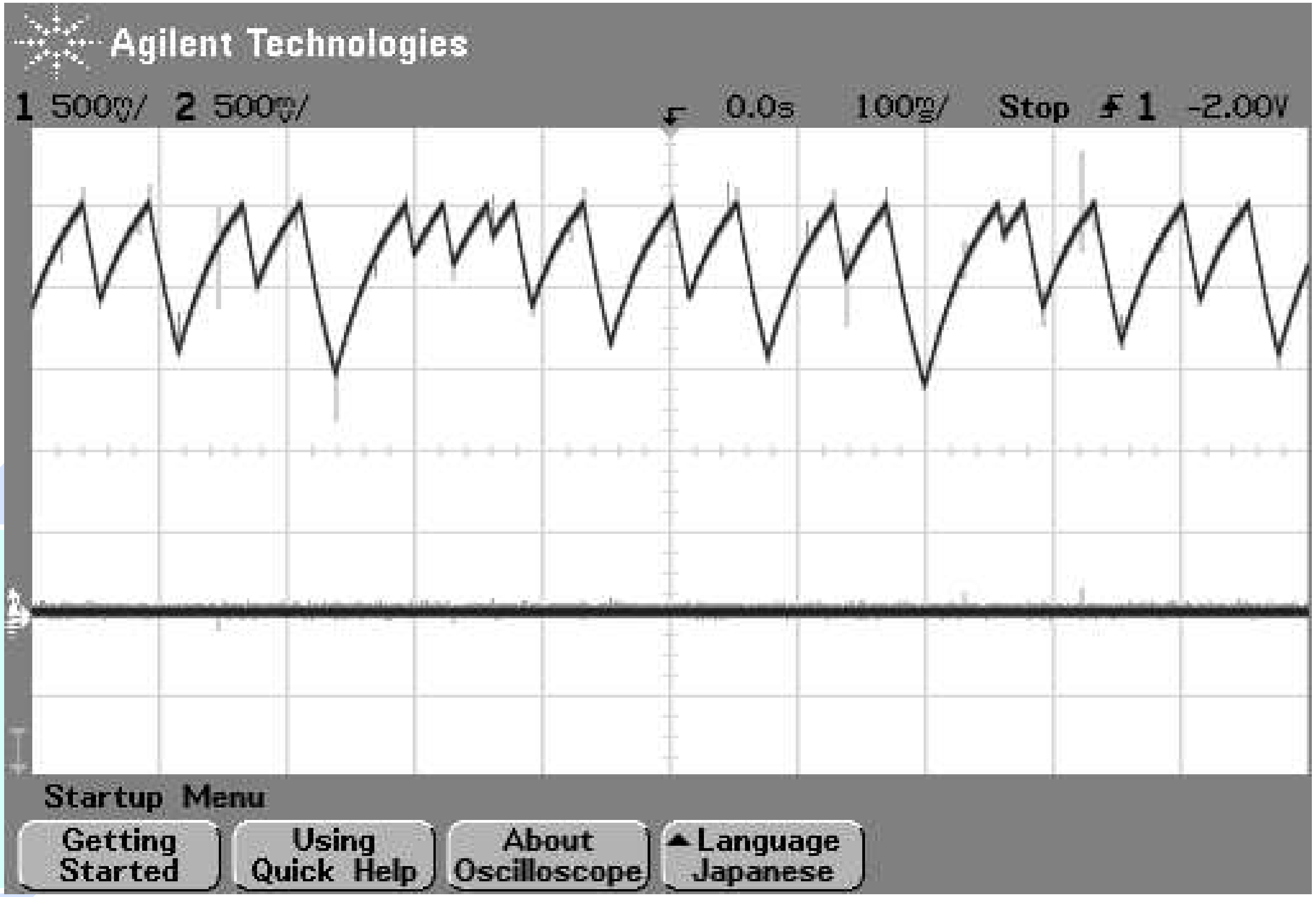


DSP implementation of PDFC



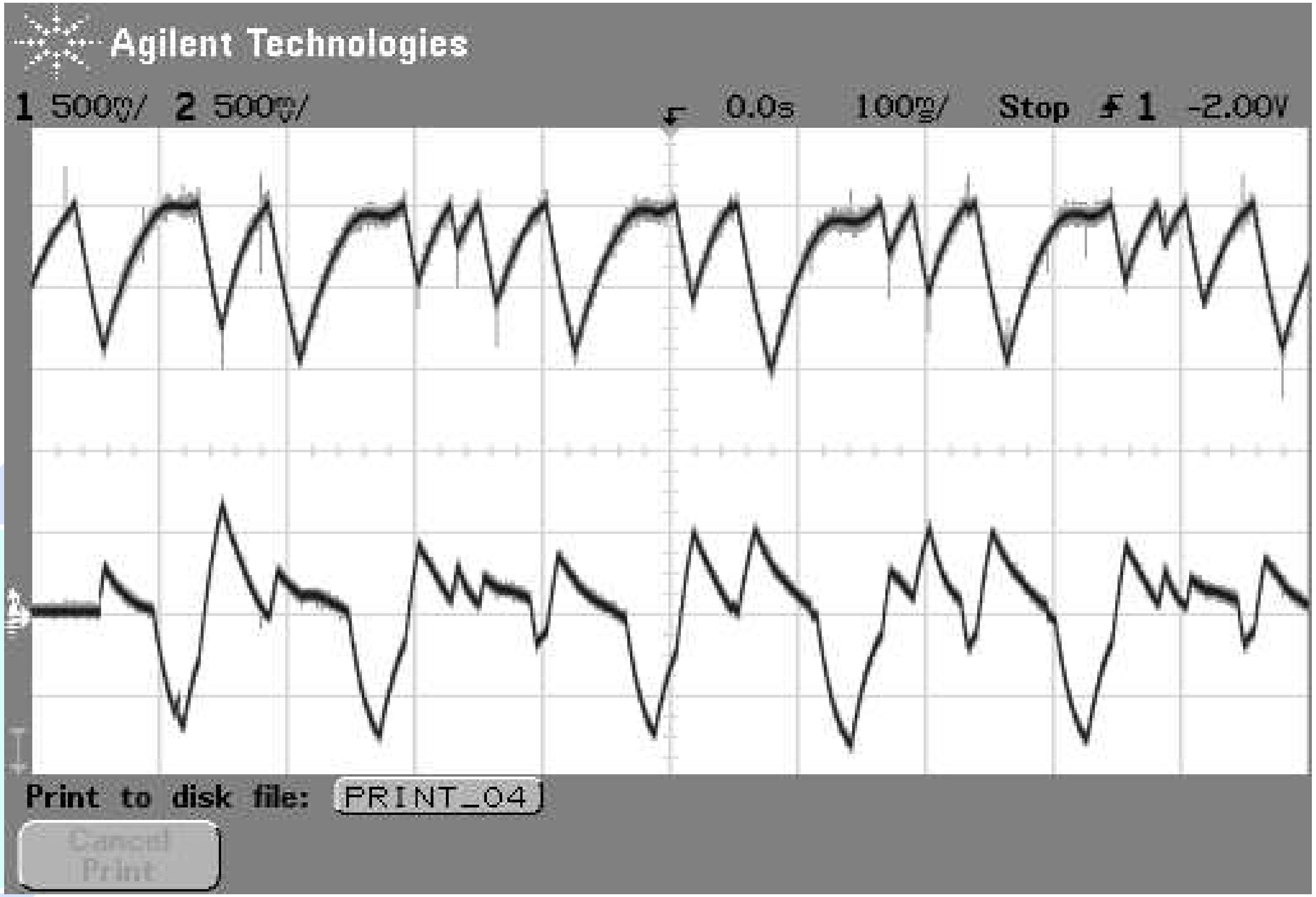
DSP implementation of PDFC

Physical measurement: $v(t)$ and $u(t)$, $K = 0.3$



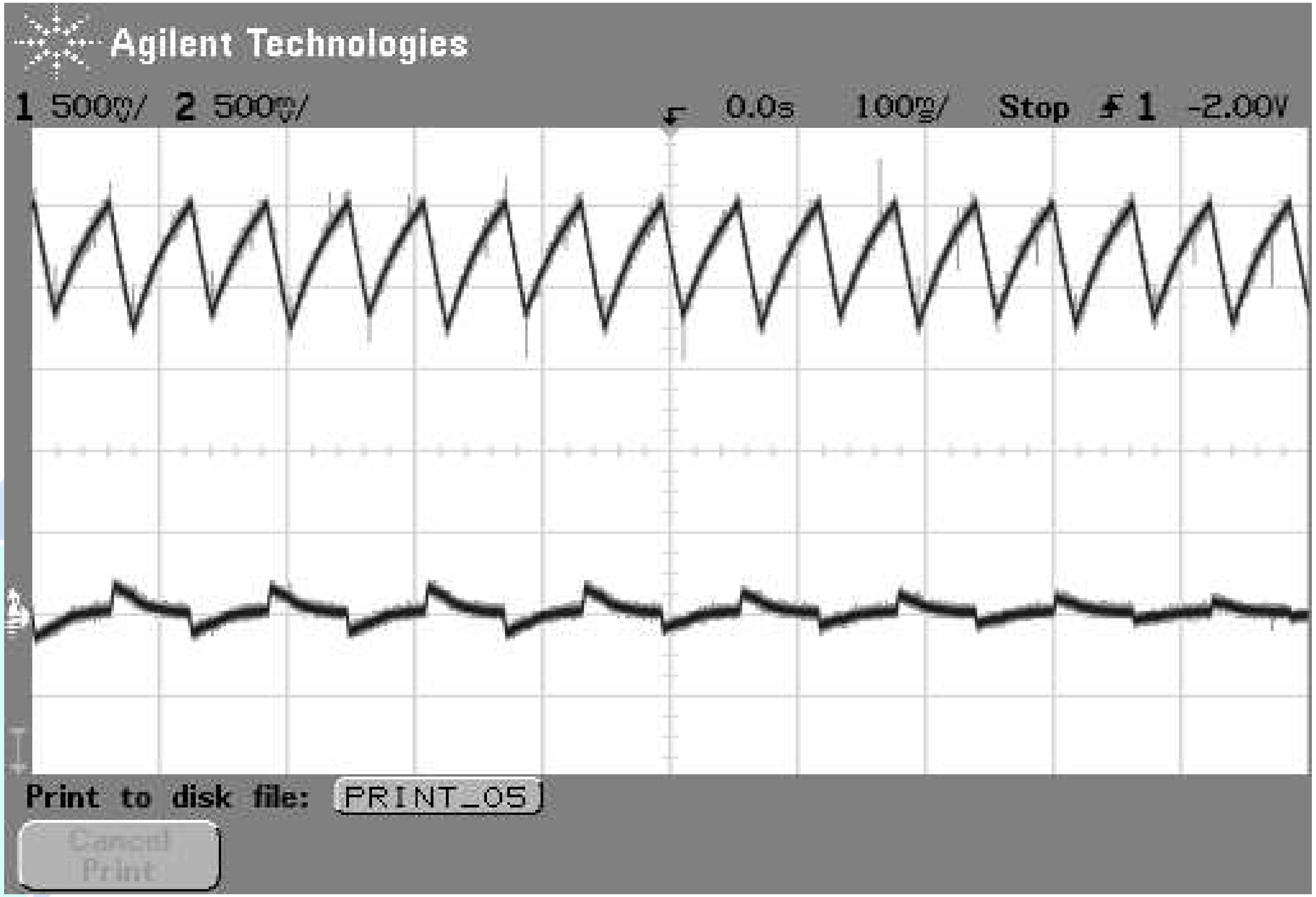
DSP implementation of PDFC

Start controlling



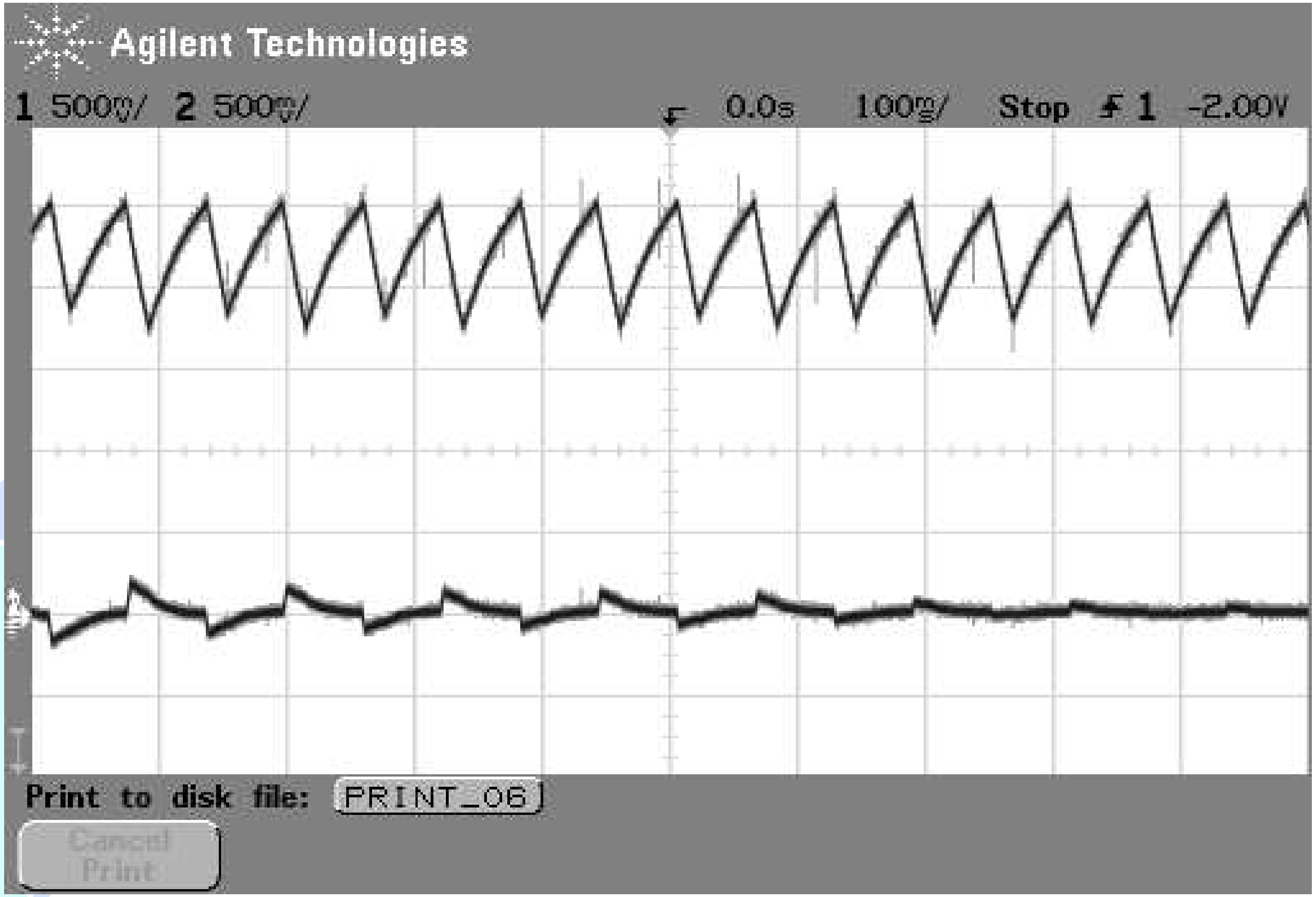
DSP implementation of PDFC

Transition



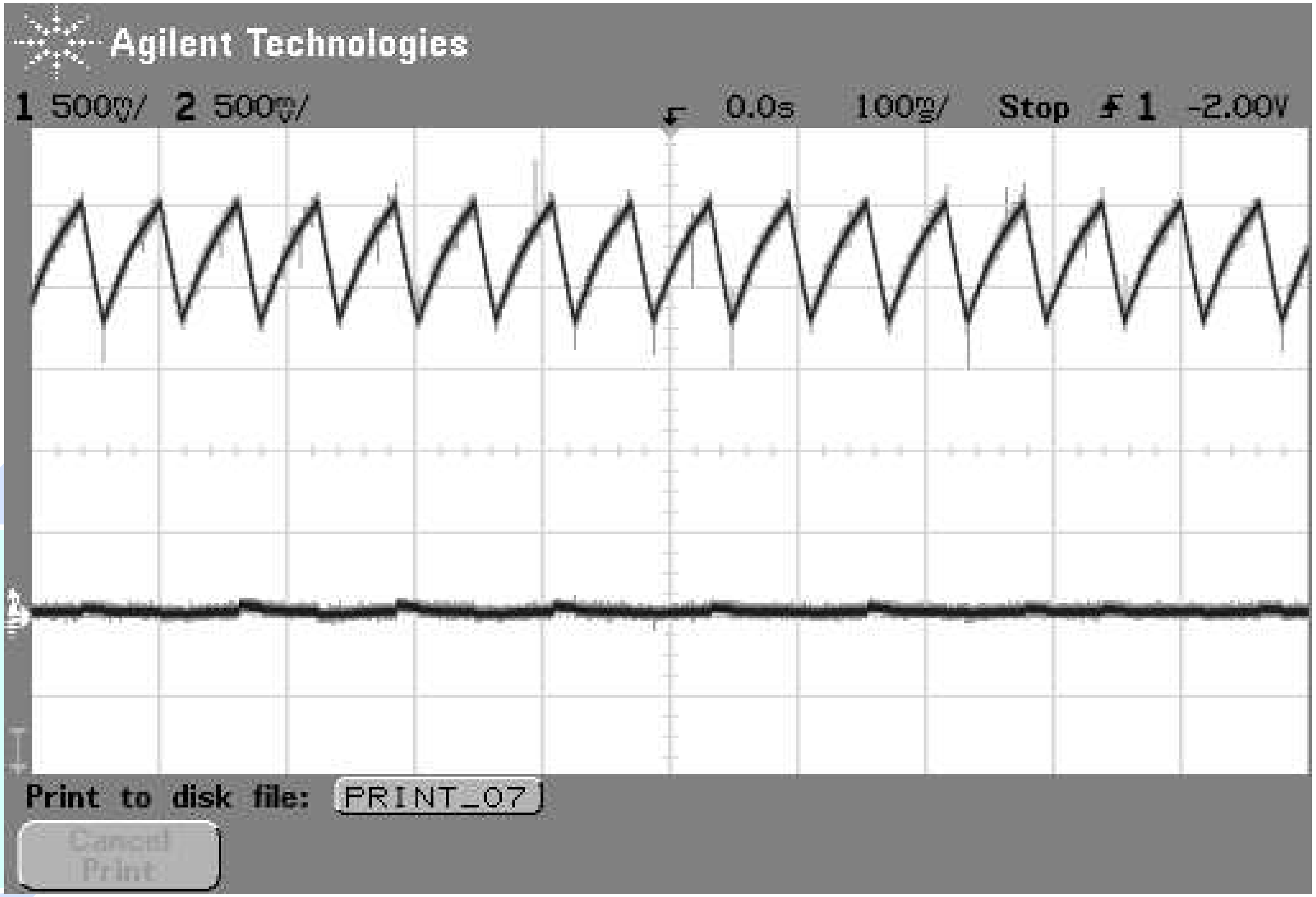
DSP implementation of PDFC

Transition



DSP implementation of PDFC

Control completed

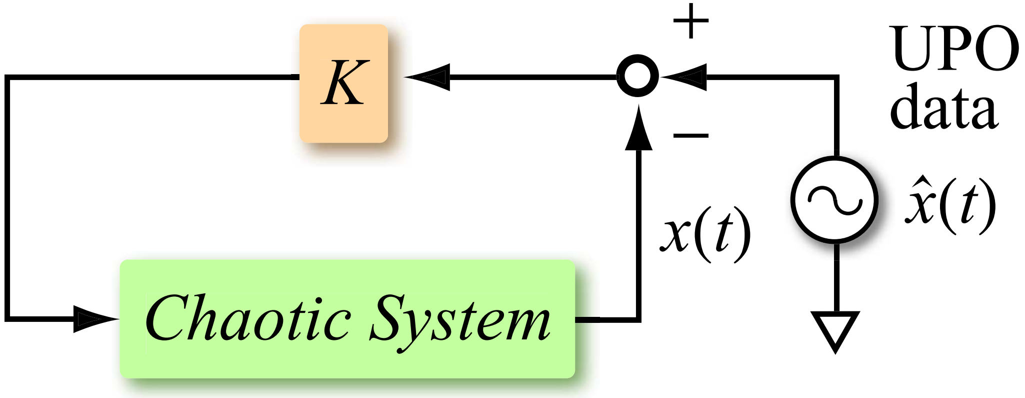


Discussion

- **Robustness is obtained.**
  (confirmed by computing a basin of attraction)
- **DSP works well for PDFC in a hybrid system.**
- **PDFC (even though DFC): The system becomes an infinite dimensional system — too difficult to analyze its stability**

Featuring EFC

External Force Control:



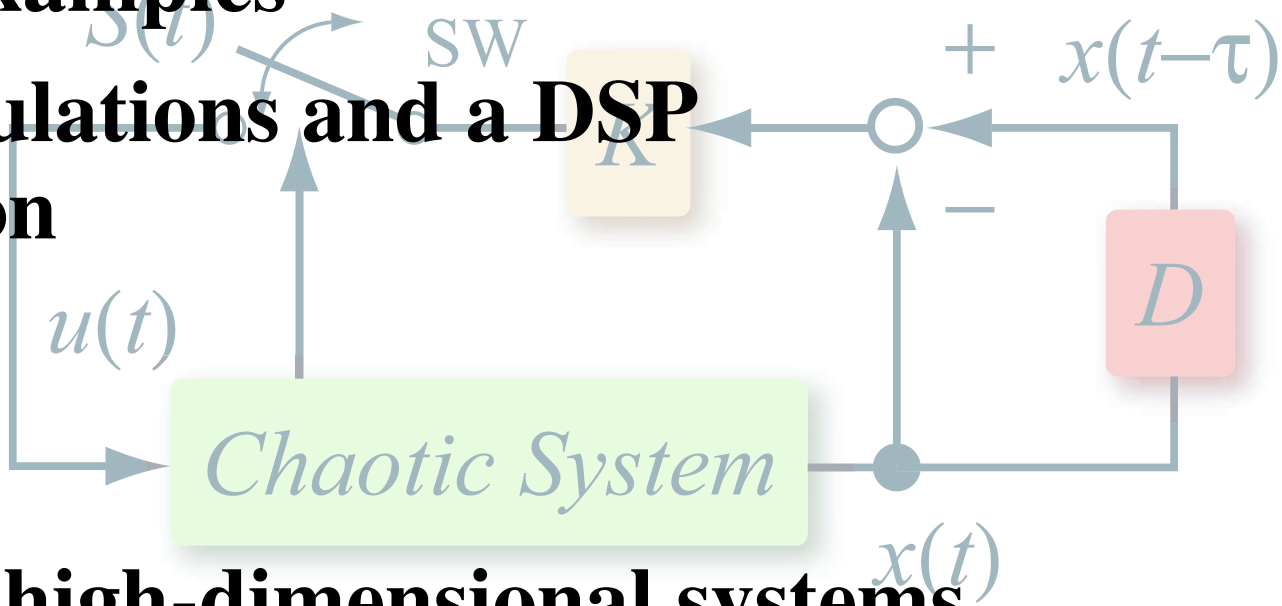
Strategy:

1. Attract the orbit near an UPO by PDFC
2. Switch to EFC (very easy to implement it by program codes)
3. Stabilize UPO by EFC
4. Stability analysis is possible!

Conclusions

PDFC—Partial Delayed Feedback Control

- concept and examples
- computer simulations and a DSP implementation
- PDFC + EFC



Future problems

- application to high-dimensional systems
- stabilizing various UPOs
- stability analysis